

Format and topics for final exam
Math 129b

General information. The final will be about twice as long as our in-class exams, with 135 minutes in which to complete it. It will take place in our usual room.

The final will be cumulative; in other words, the final will cover the topics on this sheet and also on the previous three review sheets. However, the exam will somewhat emphasize the material listed here from Chapter 8 and Chapter 4. As always, most of the exam will rely on understanding the problem sets (including the problems to be done but not written up or turned in) and the definitions and theorems that lie behind them. If you can do all of the homework, and you know and understand all of the definitions and the statements of all of the theorems we've studied, you should be in good shape. You should not spend time memorizing proofs of theorems from the book, though understanding those proofs does help you understand the theorems. On the other hand, you should definitely spend time memorizing the *statements* of the important theorems in the text.

The usual ground rules apply: No books or notes allowed, and four basic types of questions, namely, computations, statements of definitions and theorems, proofs, and true/false with justification.

Definitions. The most important definitions we have covered are:

8.1 eigenvalue	eigenvector
λ -eigenvector	λ -eigenspace
$E_T(\lambda)$	characteristic polynomial
characteristic equation	eigenstuff of a matrix
8.2 similar	similarity class
characteristic polynomial (of a linear operator)	diagonal matrix
diagonalizable	trace
8.3 basis of eigenvectors	
4.1 inner product	$\langle \cdot, \cdot \rangle$
inner product space	dot product
$\mathbb{C}([a, b])$ (continuous functions)	standard inner product on $\mathbb{C}([a, b])$
norm	$\ \mathbf{v}\ $
4.2 orthogonal vectors	
4.3 triangle inequality	angle (in an IP space)
orthogonal	
4.4 orthogonal set	normalization
unit vector	orthonormal set
8.4 symmetric matrix	orthogonal matrix
symmetric linear operator	

Examples. You will also need to be familiar with the key properties of the main examples we have discussed. Most of the important examples we have encountered have appeared in the assigned problems, both those to be turned in and those not to be turned in. In addition, you should also know:

8.1: Eigenstuff for $\frac{d}{dx}$, $\frac{d^2}{dx^2}$ (PS08).

8.2: Matrices that have the same characteristic polynomial but are not similar.

8.3: Specific examples of diagonalization (PS09); theoretical ideas behind diagonalization (PS09).

4.1: Inner product spaces: Dot product on \mathbb{R}^n , weird dot products on \mathbb{R}^n , $\int_a^b f(x)g(x) dx$ on $C([a, b])$.

4.2: Computing angles and distances in \mathbb{R}^n .

4.3: Applying Cauchy-Schwarz.

8.4: Specific examples of diagonalization of a symmetric matrix with an orthogonal matrix (PS11).

Theorems, results, algorithms. The most important theorems, results, and algorithms we have covered are listed below. You should understand all of these results, and you should be able to state any theorem clearly and precisely. You don't have to memorize theorems by number or page number; however, you should be able to state a theorem, given a reasonable identification of the theorem (either a name or a vague description).

- 8.1:** Eigenvalues are roots of characteristic polynomial; computing bases for eigenspaces of a matrix.
- 8.2:** Similarity and characteristic polynomial (Thm. 8.8).
- 8.3:** Recognizing basis eigenvectors (Thm. 8.12); diagonalizable iff there exists a basis of eigenvectors (Thm. 8.13); The Independence of Distinct Eigenstuff (TIDE) (Thm. 8.14); Diagonalization Theorem (Thms. 8.12 + 8.13 + 8.15); linear operator with n distinct eigenvalues is diagonalizable. How to diagonalize a diagonalizable matrix: finding P , finding $P^{-1}AP$ without calculation.
- 4.1:** Properties of inner products. Dot product and standard inner product on $C([a, b])$ are inner products.
- 4.2:** Law of Cosines ($\mathbf{v} \cdot \mathbf{w} = \|\mathbf{v}\| \|\mathbf{w}\| \cos \theta$).
- 4.3:** Cauchy-Schwarz inequality; triangle inequality and similar properties (Thm. 4.9).
- 4.4:** Orthogonal sets of nonzero vectors are linearly independent (Thm. 4.14); Gram-Schmidt orthonormalization theorem.
- 8.4:** Alternate descriptions of symmetric (Thm. 8.19); orthogonal change of basis preserves symmetry (Thm. 8.21); characteristic polynomial of a symmetric matrix has real roots (Thm. 8.23); Spectral Theorem, linear operator and matrix versions (Thm. 8.24 and Cor. 8.25); Orthogonality of Distinct Eigenstuff of Symmetric operators (ODES) (Thm. 8.26). How to diagonalize a symmetric matrix with an orthogonal matrix.

Not on exam. Section 4.4: You do not have to memorize the formulas for the Gram-Schmidt orthonormalization process; nor will you have to apply the process. However, you should understand the main consequences of the theorem (e.g., every finite-dimensional inner product space has an orthonormal basis). Singular value decomposition.

Good luck.