Sample exam 1
Math 129b, Fall 2004

You will not be allowed to use books, notes, or calculators. Unless otherwise stated, you may take as given anything which has been proven in class, in the homework, or in the reading.

1. (16 points) Let V be a vector space, and let $\mathbf{v}_1, \ldots, \mathbf{v}_k$ be vectors in V. Define what it means for $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ to be linearly independent.

For questions 2–5, you are given a statement. If the statement is true, you need only write "True", though a justification may earn you partial credit if the correct answer is "False". If the statement is false, write "False", and justify your answer as specifically as possible. (Do not just write "T" or "F", as you may not receive any credit; write out the entire word "True" or "False".)

2. (12 points) Let

$$S = \left\{(x, y, z) \in \mathbb{R}^3 \mid xy = z^2\right\}.$$

Then S is a subspace of $\mathbb{R}^3$.

3. (12 points) Let $S = \left\{(x, y, 0) \in \mathbb{R}^3\right\}$. The set $\{(1,0,0), (0,1,0), (0,0,1)\}$ spans S.

4. (12 points) There exists a subspace S of $\mathbb{M}(2,2)$ that contains exactly one element.

5. (12 points) Let $V = \{(v_1, v_2) \mid v_1, v_2 \in \mathbb{R}\}$, and let addition and scalar multiplication in V be defined by:

$$(v_1, v_2) + (w_1, w_2) = (v_1 + w_1 + 1, v_2 + w_2 + 1),$$
$$r(v_1, v_2) = (rv_1 + (r - 1), rv_2).$$

Then V is a vector space.

6. (16 points) Let V be vector space, and let $\mathbf{v}$ be an element of V. Carefully prove that $3\mathbf{v} + 2((-1)\mathbf{v}) = \mathbf{v}$, relying only on the axioms of a vector space. Make sure that you indicate clearly each time you use an axiom. (You do not need to cite the axioms by number, though you may do so if you like; just indicate what the axiom says or refer to it by name.)

7. (20 points) Let

$$S = \left\{f \in \mathbb{F}(\mathbb{R}) \mid f(x^2) - f(x) = 0 \text{ for all } x \in \mathbb{R}\right\}.$$

In other words, let S be the set of all $f \in \mathbb{F}(\mathbb{R})$ such that $f(x^2) - f(x) = 0$ for all $x \in \mathbb{R}$. Prove that S is a subspace of $\mathbb{F}(\mathbb{R})$.