

Math 131A, problem set 04
Outline due: Wed Sep 24
Completed version due: Mon Sep 29
Last revision due: Mon Oct 20

All problem numbers refer to Bartle and Sherbert.

Problems to be done but not turned in: (3.1) 1, 3, 5, 6, 9, 11, 13, 15; (3.2) 1, 3.

Problems to be turned in:

1. Let a be a real number, and let $x_n = a$ be a constant sequence. Prove that $\lim(x_n) = a$, using the definition of limit.
2. Find $\lim \left(\frac{7n + 13}{2n - 5} \right)$. Prove your answer, using the definition of limit.
3. Consider the sequence $\left(\frac{n!}{n^n} \right)$.
 - (a) Find a sequence x_n such that $\lim x_n = 0$ and for n larger than some fixed integer (e.g., $n \geq 5$), $x_n \geq \frac{n!}{n^n}$.
 - (b) Find $\lim \left(\frac{n!}{n^n} \right)$, and prove your answer, using the definition of limit.
4. Prove that if (x_n) and (y_n) are sequences and M is a real number such that $\lim(x_n) = 0$ and $|y_n| \leq M$ for all $n \in \mathbf{N}$, then $\lim(x_n y_n) = 0$.
5.
 - (a) Find a sequence (x_n) such that (x_n) diverges, but $(|x_n|)$ converges to 7. (You do not have to prove that your sequences diverge or converge to the limits you claim.)
 - (b) Find sequences (x_n) and (y_n) such that (x_n) and (y_n) diverge but $\lim(x_n + y_n) = -2$. (You do not have to prove that your sequences diverge or converge to the limits you claim.)
 - (c) Find sequences (x_n) and (y_n) such that (x_n) and (y_n) diverge but $\lim(x_n y_n) = 3$. (You do not have to prove that your sequences diverge or converge to the limits you claim.)
6. (3.1) 17.
7. Let A be a bounded nonempty subset of \mathbf{R} , and let $u = \sup A$. Prove that there exists a sequence (x_n) such that $x_n \in A$ for all $n \in \mathbf{N}$ and $\lim(x_n) = u$. (Suggestion: Use the Arbitrarily Close Criterion and $\lim(1/n)$).