Math 131A, problem set 05
Outline due: Wed Oct 01
Completed version due: Mon Oct 06
Last revision due: Mon Oct 20

All problem numbers refer to Bartle and Sherbert.

Problems to be done but not turned in: (3.2) 8, 9, 11, 13, 15, 18, 20, 21, 22, 23; (3.3) 1, 11.

Problems to be turned in: Throughout this problem set, you may use any result from 3.1–3.2 you find helpful (e.g., the limit laws). For problems 1–3, compute the value of the indicated limit, and prove your answer.

1. $\lim \left(\dfrac{n^2}{n+1} - \dfrac{n^2+1}{n} \right)$.
2. $\lim \left(\dfrac{3^n + (-2)^n}{3^{n+1} + (-2)^{n+1}} \right)$.
3. $\lim \left(\dfrac{n^{3/2} \sin(n!)}{n^2 - 5} \right)$.
4. (3.2) 14. (Suggestion: Use the Squeeze Theorem.)
5. (3.2) 19.
6. Let (x_n) be a bounded sequence, and for each $n \in \mathbf{N}$, let

$$\begin{aligned} T_n &= \{x_k \mid k \geq n\}, \\ s_n &= \sup T_n = \sup \{x_k \mid k \geq n\}, \\ \ell_n &= \inf T_n = \inf \{x_k \mid k \geq n\}. \end{aligned}$$

   Note that in the following, we do not assume that (x_n) is either increasing nor decreasing (though that may happen to be the case).

   (a) Prove that s_2 is an upper bound of T_{37}.

   (b) Let u be an upper bound of (x_n). Prove that for any $n \in \mathbf{N}$, $\ell_n \leq u$.

   (c) Prove that (s_n) is a decreasing sequence, and that (ℓ_n) is an increasing sequence. (Suggestion: Use the definitions of sup and inf.)

   (d) Prove that (s_n) and (ℓ_n) each converge. (The limit $\lim s_n$ is called the **lim sup** of (x_n), and the limit $\lim \ell_n$ is called the **lim inf** of (x_n).)