Math 131A, problem set 08
Outline due: Wed Oct 29
Completed version due: Mon Nov 03
Last revision due: Wed Nov 19

All problem numbers refer to Bartle and Sherbert.
Problems to be done but not turned in: (4.1) 1, 2, 5, 8, 9, 10, 11, 13, 14, 16.
Problems to be turned in:

1. Use the definition of limit to prove that $\lim_{x\to 3} \frac{x+5}{x-2} = 8$.

2. Use the definition of limit to prove that $\lim_{x\to c} |x| = |c|$. (You may want to handle the cases $c > 0$, $c = 0$, and $c < 0$ separately.)

3. Let $f : \mathbf{R} \to \mathbf{R}$ be defined by

$$f(x) = \begin{cases} x\sin(1/x) & \text{for } x \neq 0, \\ 0 & \text{for } x = 0. \end{cases}$$

   Use the definition of limit to prove that $\lim_{x\to 0} f(x) = 0$.

4. Prove that $\lim_{x\to 0} \frac{1}{\sqrt[3]{x}}$ does not exist.

5. Let $g : \mathbf{R} \to \mathbf{R}$ be defined by

$$g(x) = \begin{cases} 1 & \text{for } x \in \mathbf{Q}, \\ 0 & \text{for } x \notin \mathbf{Q}. \end{cases}$$

   Prove that for any $c \in \mathbf{R}$, $\lim_{x\to c} g(x)$ does not exist.

6. Let I be an interval, let $f : I \to \mathbf{R}$ be a function, and let $c \in I$ and $\delta > 0$ be such that $V = V_\delta(c)$ is contained in I. Let $f_1 : V \to \mathbf{R}$ be defined by $f_1(x) = f(x)$ (i.e., let f_1 be the restriction of f to the neighborhood V). Prove if $\lim_{x\to c} f_1(x) = L$, then $\lim_{x\to c} f(x) = L$ (in particular, the limit exists).

7. (4.1) 12.