Math 131A, problem set 09
Outline due: Wed Nov 05
Completed version due: Mon Nov 10
Last revision due: Wed Dec 10

All problem numbers refer to Bartle and Sherbert.
Problems to be done but not turned in: (4.2) 1, 2, 3, 4, 8, 11; (5.1) 4, 7, 8, 10.
Problems to be turned in:

1. Let $g : \mathbf{R} \to \mathbf{R}$ be defined by
$$g(x) = \begin{cases} x & \text{for } x \in \mathbf{Q}, \\ 0 & \text{for } x \notin \mathbf{Q}. \end{cases}$$
Prove that $\lim_{x \to 0} g(x) = 0$. You may use the results of 4.2.

2. Let $f(x) = x^3$. Carefully use the limit laws to evaluate $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$. In particular, indicate clearly where (if anywhere) you use $h \neq 0$.

3. (4.2) 7.

4. Use the ϵ-δ definition of continuity directly to prove that x^3 is continuous at $x = 2$.

5. Let $g : \mathbf{R} \to \mathbf{R}$ be defined by
$$g(x) = \begin{cases} x & \text{for } x \in \mathbf{Q}, \\ 0 & \text{for } x \notin \mathbf{Q}. \end{cases}$$
Prove that $g(x)$ is continuous at 0 and discontinuous at every $c \in \mathbf{R}$ such that $c \neq 0$. You may use any of the definitions and results of 5.1.

6. (5.1) 1.

7. Let $f : \mathbf{R} \to \mathbf{R}$ be a function such that $f(x + y) = f(x) + f(y)$ for all $x, y \in \mathbf{R}$, and suppose that $\lim_{x \to 0} f(x) = 0$. Prove that $f(x)$ is continuous at every $c \in \mathbf{R}$. (Suggestion: First prove that $f(0) = 0$ and $f(-a) = -f(a)$ for all $a \in \mathbf{R}$.)