

Math 131A, problem set 10
Outline due: Wed Nov 12
Completed version due: Mon Nov 17
Last revision due: Wed Dec 10

All problem numbers refer to Bartle and Sherbert.

Problems to be done but not turned in: (5.1) 13; (5.2) 1, 2, 4, 7, 12, 13, 15; (5.3) 1, 3.

Problems to be turned in:

1. Let $f : [0, 1] \rightarrow \mathbf{R}$ and $g : [-1, 0) \rightarrow \mathbf{R}$ be functions, and let $h : [-1, 1] \rightarrow \mathbf{R}$ be defined by

$$h(x) = \begin{cases} g(x) & \text{for } x < 0, \\ f(x) & \text{for } x \geq 0. \end{cases}$$

Suppose that $\lim_{x \rightarrow 0} g(x)$ exists. Prove that $h(x)$ is continuous at 0 if and only if $f(x)$ is continuous at $x = 0$ and $\lim_{x \rightarrow 0} g(x) = f(0)$.

2. (5.2) 3.
3. (5.2) 5.
4. (5.2) 6.
5. Let $h : \mathbf{R} \rightarrow \mathbf{R}$ be a continuous function on \mathbf{R} such that $h(r) = 0$ for all $r \in \mathbf{Q}$. Prove that $h(x) = 0$ for all $x \in \mathbf{R}$.
6. (a) Give an example of a continuous function $f : [0, \infty) \rightarrow \mathbf{R}$ such that f has neither an absolute minimum nor an absolute maximum on $[0, \infty)$. Prove your answer.
- (b) Let $I = (0, 1] = \{x \in \mathbf{R} \mid 0 < x \leq 1\}$. Give an example of a continuous function $f : I \rightarrow \mathbf{R}$ such that f has neither an absolute minimum nor an absolute maximum on I . Prove your answer. (For partial credit, try this for $I = (0, 1)$.)
7. (5.3) 2.