Format and topics
Exam 1, Math 131a

General information. Exam 1 will be a timed test of 50 minutes, covering 1.2, 2.1–2.5, and 3.1 of the text. No books, notes, calculators, etc., are allowed. Most of the exam will rely on understanding the problem sets and the definitions and theorems that lie behind them. If you can do all of the homework, and you know and understand all of the definitions and the statements of all of the theorems we've studied, you should be in good shape.

You should not spend time memorizing proofs of theorems from the book, though understanding those proofs does help you understand the theorems. On the other hand, you should defintely spend time memorizing the *statements* of the important theorems in the text.

Types of questions. There are four types of questions that may appear on exams in this class, namely:

1. Computations;
2. Statements of definitions and theorems;
3. Proofs;
4. True/false with justification.

Computations. These will be drawn from computations of the type you've done on the problem sets. You do not need to explain your answer on a computational problem, but show all your work.

Statements of definitions and theorems. In these questions, you will be asked to recite a definition or the statement of a theorem from the book. You will not be asked to recite the proofs of any theorems from the book, though you may be asked to prove book theorems that you might have been asked to prove on problem sets.

Proofs. These will resemble some of the shorter problems from your homework. You may take as given anything that has been proven in class, in the homework, or in the reading. Partial credit may be given on proof questions, so keep trying if you get stuck (and you've finished everything else). If all else fails, at least try to write down the definitions of the objects involved.

True/false with justification. This type of question may be less familiar. You are given a statement, such as:

- For $a, b \in \mathbb{R}$, if $a \geq b$, then $-a \geq -b$.

If the statement is true, all you have to do is write "True". (However, see below.) If the statement is false (like the one above), not only do you have to write "False", but you must also give a reason why the statement is false. Your reason might be a very specific counterexample:

> False. We have $3 \geq 2$, but $-3 < -2$, which means that $-3 \not\geq -2$.

Your reason might also be a more general principle:

> False. For any $a, b \in \mathbb{R}$, if $a > b$, then $-a < -b$, which means that $-a \not\geq -b$.

Either way, your answer should be as specific as possible to ensure full credit.

Depending on the problem, some partial credit may be given if you write "False" but provide no justification, or if you write "False" but provide insufficient or incorrect justification. Partial credit may also be given if you write "True" for a false statement, but provide some partially reasonable justification. (In other words, if you have time, it can't hurt to justify "True" answers.)

If I can't tell whether you wrote "True" or "False", you will receive no credit. In particular, please do not just write "T" or "F", as you may not receive any credit.

Definitions. The most important definitions we have covered are:

1.2	induction hypothesis	base case
2.1	irrational number	positive number
	$a > 0$, $a \geq 0$	$a > b$, $a \geq b$
2.2	absolute value, $\lvert a \rvert$	ϵ-neighborhood $V_\epsilon(a)$
	distance $\lvert x - y \rvert$	
2.3	bounded above/below	upper/lower bound
	bounded	unbounded
	supremum, $\sup S$	least upper bound
	infimum, $\inf S$	greatest lower bound
2.5	open interval	endpoints (of an interval)
	closed interval	half-open, half-closed
	length (of an interval)	infinite open interval
	infinite closed interval	nested (intervals)
3.1	sequence of real numbers	sequence in $\mathbb{R}$
	term, element	$\lim(x_n)$
	$x_n \to x$	inductively defined
	constant sequence	converge
	limit	convergent
	divergent	m-tail

Examples. You will also need to be familiar with the key properties of the main examples we have discussed. The most important examples we have seen are:

1.2: Sample induction proofs, geometric series.
2.3: Bounded sets that do and do not contain their sup/inf.
2.4: Examples of rational and irrational numbers (e.g., $\sqrt{2}$) and related phenomena.
3.1: Constant sequence, $\left(\frac{1}{n}\right)$, $(0, 2, 0, 2, \ldots)$; if $0 \leq b < 1$, $\lim(b^n) = 0$. Examples of ϵ-$K(\epsilon)$ procedure (Examples 3.1.6, 3.1.11).

Theorems, results, algorithms. The most important theorems, results, and algorithms we have covered are listed below. You should understand all of these results, and you should be able to state any theorem clearly and precisely. You don't have to memorize theorems by number or page number; however, you should be able to state a theorem, given a reasonable identification of the theorem (either a name or a vague description).

1.2: Well-ordering property of $\mathbb{N}$; Principle of mathematical induction.
2.1: There is no rational whose square is 2. Order axioms of $\mathbb{R}$; basic consequences (Thms. 2.1.7–2.1.11), especially multiplying both sides of an inequality (2.1.7(c)) and No Smallest Positive Number Trick (Thm. 2.1.9).
2.2: Basic properties of $|a|$ (esp. $|ab| = |a|\,|b|$ and $|a| \leq c$ if and only if $-c \leq a \leq c$). Triangle inequality and variations. Idea: $|x - y|$ is distance between x and y.
2.3: Completeness Axiom/Supremum Property. Arbitrarily Close Criterion (Lem. 2.3.4).
2.4: Archimedean property (2.4.3–2.4.6). $\sqrt{2}$ is real. Rationals are not complete. Density of rationals/irrationals in $\mathbb{R}$.
2.5: Nested Intervals Property. Real numbers are uncountable.
3.1: Uniqueness of Limits. Equivalent conditions for existence of a limit. The Tails Theorem (Thm. 3.1.9).

Not on exam. (2.1) Field axioms. (2.5) Characterization of Intervals; Binary/decimal representations, periodic decimals, Cantor's second proof (pp. 48–50).

Other. You should have a working familiarity with the techniques and strategies for proof and logic tips from the handout on "What is a proof?" You do not need to memorize information from the handout, but you do need to be able to apply it.

Good luck.