Format and topics for exam 3
Math 131a

General information. Exam 3 will cover 3.7, 4.1–4.2, and 5.1–5.4 of the text (or roughly speaking, PS07–PS11). The exam will be cumulative only to the extent that 3.7, 4.1–4.2, and 5.1–5.4 rely on previous material; for example, you should still know what the supremum of a subset of $\mathbb{R}$ is. However, there will not be any questions on the exam that only cover old material; for example, you will not be asked to define the supremum of a subset of $\mathbb{R}$.

As before, most of the exam will rely on understanding the problem sets and the definitions and theorems that lie behind them. If you can do all of the homework, and you know and understand all of the definitions and the statements of all of the theorems we've studied, you should be in good shape. You should not spend time memorizing proofs of theorems from the book, though understanding those proofs may help you understand the theorems. On the other hand, you should defintely spend time memorizing the *statements* of the important theorems in the text.

Exam 3 will follow the same ground rules as before.

Definitions. The most important definitions we have covered are:

3.7	infinite series generated by (x_n)	terms of a series
	partial sums of a series	convergent series
	sum of a series	divergent series
	geometric series	p-series
	harmonic series	alternating harmonic series
4.1	cluster point	limit of f at c
	$\lim\limits_{x\to c} f(x)$	f converges to L at c
	f diverges at c	limit does not exist
4.2	bounded on a neighborhood of c	$f+g$, fg, etc.
5.1	continuous at c	discontinuous at c
	continuous on the set B	
5.3	f bounded on A	f unbounded on A
	absolute minimum on A	absolute maximum on A
	absolute minimum point	absolute maximum point
5.4	uniformly continuous	

Examples. You will also need to be familiar with the key properties of the main examples we have discussed. Most of the important examples we have encountered have appeared in the assigned problems. In addition, you should also know:

3.7: Geometric series, harmonic series, p-series, alternating harmonic series. Using definition of sum of a series to prove convergence/divergence (HW).

4.1: Cluster points of various sets, esp. $\{1/n \mid n \in \mathbb{N}\}$, all rationals in $[0, 1]$. Signum function, $\sin(1/x)$. Using ϵ-δ definition to prove limits (Ex. 4.1.7). Using Sequential Criterion to prove limits (HW); using Sequential Criterion to prove divergence.

4.2: Using Combination Theorems to prove limits (Ex. 4.2.5).

5.1: Constant functions, x, x^2, $1/x$, signum, Dirichlet's discontinuous function, Thomae's function. Examples of using ϵ-δ definition to prove continuity (Ex. 5.1.6); examples of using Sequential Criterion to prove continuity (HW).

5.2: Polynomials, rationals, trig functions.

5.3: Examples that show all hypotheses of Max-Min and Boundedness Theorems are necessary.

5.4: Functions that are continuous but not uniformly continuous.

Theorems, results, algorithms. The most important theorems, results, and algorithms we have covered are listed below. You should understand all of these results, and you should be able to state any theorem clearly and precisely. You don't have to memorize theorems by number or page number; however, you should be able to state a theorem, given a reasonable identification of the theorem (either a name or a vague description).

3.7: nth Term Test (3.7.3). Cauchy Criterion for Series (3.7.4). Convergence of bounded positive series (Thm. 3.7.5) Comparison Test (3.7.7); Limit Comparison Test (3.7.8). Ratio Test for positive series (notes).

4.1: Alternate version of cluster points (Thm. 4.1.2). Limits are unique (Thm. 4.1.5), neighborhood version of limit defn. (Thm. 4.1.6). Sequential Criterion for Limits (Thm. 4.1.8). Divergence Criteria (Thm. 4.1.9).

4.2: Limit implies bounded in a neighborhood (4.2.2). Combination Theorems for Limits. Pre-Squeeze Theorem (4.2.6); Squeeze Theorem (4.2.7). Positive (resp. negative) limit implies positive (resp. negative) nearby (Thm. 4.2.9).

5.1: Sequential Criterion for Continuity (5.1.3). Discontinuity Criteria (5.1.4).

5.2: Combination Theorems for continuity (Thm. 5.2.1). Continuity of absolute values, square roots (5.2.4—5.2.5). Composition of continuous is continuous (Thm. 5.2.6).

5.3: Boundedness Theorem. Maximum-Minimum Theorem. Location of Roots Theorem. Bolzano's Intermediate Value Theorem.

5.4: Criteria for nonuniform continuity. Uniform Continuity Theorem.

Not on exam. (5.3) Preservation of Intervals Theorem. (5.4) Lipschitz functions; Lipschitz implies uniform continuity; Continuous Extension Theorem; Approximation.

Good luck.