

**Sample Exam 1**  
**Math 131A, Fall 2008**

This exam (from Spring 2005) covered all of Chapters 1 and 2. Our exam will have much less material from Chapter 1 (Section 1.2 only) and will include section 3.1, so this is more of an indication of format than an indication of subject material.

1. (16 points) Let  $S$  be a nonempty subset of  $\mathbb{R}$ . Define what it means for a real number  $w$  to be an infimum of  $S$ , i.e., define what it means to say that  $w = \inf S$ .
2. (14 points) Find all  $x \in \mathbb{R}$  such that

$$(x - 1)(2 - |x - 1|) > 0.$$

No explanation necessary, but show all your work.

For questions 3–5, you are given a statement. If the statement is true, you need only write “True”, though a justification may earn you partial credit if the correct answer is “False”. If the statement is false, write “False”, and justify your answer **as specifically as possible**. (Do not just write “T” or “F”, as you may not receive any credit; write out the entire word “True” or “False”.)

3. (12 points) Let  $S$  be a nonempty set of the rational numbers  $\mathbb{Q}$ . If  $S$  is bounded above, then it must be the case that  $S$  has a supremum in the rational numbers  $\mathbb{Q}$ .
4. (12 points) If  $S$  is a nonempty subset of the real numbers that is bounded above, then  $S$  must have an upper bound in the natural numbers (i.e., there must exist a natural number  $n$  such that  $n$  is an upper bound for  $S$ ).
5. (12 points) Let  $a$  and  $b$  be real numbers such that  $a < b$ . It is possible that  $b$  is contained in the  $\epsilon$ -neighborhood of  $a$  for every  $\epsilon > 0$ .
6. (16 points) **PROOF QUESTION.** Prove by induction that  $n^2 > 2n + 3$  for all natural numbers  $n$  such that  $n \geq 4$ .
7. (18 points) Let  $X = \{x \in \mathbb{R} \mid x \geq 0\}$  (i.e., let  $X$  be the set of all nonnegative real numbers), and let  $f : X \rightarrow X$  be defined by the formula

$$f(x) = \sqrt{x}.$$

Prove that  $f$  is a bijection. (Note: This material will not appear on our test, but our test will probably have two proof problems on it.)