Sample Exam 2
Math 131A, Fall 2008

This exam (from Spring 2005) covered 3.1–3.5. Our exam will also include 3.7.

1. (20 points) The Bolzano-Weierstrass Theorem says:

"Every (a) _____ (b) _____

has a (c) _____ (d) _____."

- Find single words (a), (b), (c), and (d) that complete the statement of Bolzano-Weierstrass, choosing your phrasing so that (d) is a noun.
- Define the word (d). (Reminder: For full credit, (d) must be a noun.)

For questions 2–3, you are given a statement. If the statement is true, you need only write "True", though a justification may earn you partial credit if the correct answer is "False". If the statement is false, write "False", and justify your answer **as specifically as possible**. (Do not just write "T" or "F", as you may not receive any credit; write out the entire word "True" or "False".)

2. (12 points) It is possible to find a divergent sequence (x_n) such that $x_n \leq x_{n+1}$ and $x_n \leq 3$ for all natural numbers n.

3. (12 points) Let X and Y be sequences. If $X + Y$ converges, then it must be the case that both X and Y converge.

4. (18 points) **PROOF QUESTION.** Use the definition of the limit of a sequence to prove that

$$\lim \frac{n^2+1}{n^2+2} = 1.$$

For full credit, do not use the theorems from 3.2.

5. (18 points) **PROOF QUESTION.** Consider the sequence

$$x_n = \frac{2 + \sin^2(3n-7)}{n^2}.$$

Does (x_n) converge or diverge? If (x_n) converges, find $\lim x_n$ and prove your assertion; if (x_n) diverges, prove it diverges. You may use all of the results and theorems from sections 3.1 and 3.2.

6. (20 points) Suppose that X and Y are sequences, and that

$$\lim x_n = 3, \qquad \lim y_n = 4.$$

Prove, using the definition of the limit, and not the theorems of section 3.2, that there exists a natural number M such that for all $n \geq M$,

$$x_n y_n > 6.$$

Again, for full credit, do not rely on the theorems of section 3.2.