Sample Exam 3
Math 131A, Fall 2008

This exam (from Spring 2005) covered 3.7, 4.1–4.2, and 5.1–5.2. Our exam will also include 5.3–5.4. Our exam will probably be more difficult, as it will cover more material.

1. (22 points) Let A be a subset of $\mathbb{R}$, let $f : A \to \mathbb{R}$ be a function with domain A, let $c \in A$ be a cluster point of A, and let L be a real number.

(a) Define what it means to say that $\lim_{x \to c} f(x) = L$.

(b) Define what it means for $f(x)$ to be continuous at c.

(c) Briefly, but as precisely as possible, explain the relationship between the limit of $f(x)$ at c and the continuity of $f(x)$ at c.

Questions 2–3: True/False, with the usual rules.

2. (12 points) Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that $f(x)$ is continuous at $c = 3$ and $f(3) = -4$. Let $\sum_{n=1}^{\infty} a_n$ be an infinite series with sequence of partial sums (s_n), and suppose that $\sum_{n=1}^{\infty} a_n = 3$. It is possible that $\lim_{n \to \infty} f(s_n) = 7$.

3. (12 points) It is possible to find a function $f : \mathbb{R} \to \mathbb{R}$ such that $-2 \leq f(x) \leq 3$ for all $x \in \mathbb{R}$ and $f(x)$ is continuous at all $c \in \mathbb{R}$ **except** $c = 5$.

4. (16 points) **PROOF QUESTION.** Use either the definition of the limit or the Sequential Criterion for Limits to prove that $\lim_{x \to 1} \dfrac{x - 5}{x - 3} = 2$.

5. (18 points) **PROOF QUESTION.** Let $\sum_{n=1}^{\infty} a_n$ be a convergent infinite series. Define new terms b_n for all $n \in \mathbb{N}$ by the formulas $b_1 = 86$, and $b_n = a_{n-1}$ for $n \geq 2$ (i.e., $b_1 = 86$, $b_2 = a_1$, $b_3 = a_2$, and so on). Let (s_n) be the sequence of partial sums of $\sum_{n=1}^{\infty} a_n$, and let (t_n) be the sequence of partial sums of $\sum_{n=1}^{\infty} b_n$.

Use the sequences of partial sums of $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ to prove that $\sum_{n=1}^{\infty} b_n$ converges.

6. (20 points) **PROOF QUESTION.** Let $A = \mathbb{R} \setminus \{0\}$, and let $f : A \to \mathbb{R}$ be given by

$$f(x) = \begin{cases} 1 + x \sin\left(\dfrac{1}{x}\right) & \text{for } x > 0, \\ 1 + 2x & \text{for } x < 0. \end{cases}$$

Prove that $\lim_{x \to 0} f(x) = 1$.