

**Sample Exam 2**  
**Math 19, Fall 2009**

Note that you should not think of the topic coverage from this exam as exhaustive; the exam is supplied just to give you an idea of style of question. The emphasis of your exam is likely to be quite different than the emphasis of this sample exam, as there is plenty of material that we have covered this semester that was not covered by this sample exam, and vice versa. Note also that several questions have been cut, so the exam is much shorter than yours will be. In general, your best guides to what will be covered by this exam are the homework and the list of review topics.

1. Sketch the graph of a rational function  $R(x)$  such that:

- The graph of  $R(x)$  has vertical asymptotes  $x = -1$  and  $x = 4$ .
- The graph of  $R(x)$  has a horizontal asymptote  $y = -2$ .
- The graph of  $R(x)$  has **no**  $x$ -intercepts.

You do **not** have to find a formula for  $R(x)$ . Show all your work.

2. Find a polynomial  $f(x)$  such that  $f(x)$  has degree 4, a zero of multiplicity 3 at  $x = -7$ , a zero of multiplicity 1 at  $x = 5$ , and no other zeros. No explanation necessary. Do not multiply out your final answer.

3. The company Mega-Online Used Snowmobiles Extraordinaire has found that when it charges  $p$  dollars per snowmobile, the total amount of revenue  $R$  (in dollars) that they make is

$$R(p) = -\frac{1}{3}p^2 + 2700p.$$

- (a) Graph  $R(p)$ . Make sure you clearly indicate all intercepts and points of interest on the graph.
- (b) If M.O.U.S.E. wants to maximize its revenue, what price should it charge per snowmobile? Show all your work, and state your final answer in the form of a **complete sentence**, using the correct units. Make sure you clearly answer **precisely** the question that was asked.

4. Let  $R(x) = -3\frac{(x+1)}{(x-30)(x+20)} = \frac{-3x-3}{x^2-10x-600}$ .

- (a) Find the  $y$ -intercept of  $R(x)$ .
- (b) List each real zero of  $R(x)$ , and for each zero, determine if the graph cuts the  $x$ -axis, bumps the  $x$ -axis, or slides through the  $x$ -axis.
- (c) Find all vertical asymptotes of  $R(x)$ .
- (d) Find all horizontal and oblique asymptotes of  $R(x)$ .
- (e) Indicate the  $x$  values where  $R(x) > 0$  and the  $x$  values where  $R(x) < 0$ . Show all your work.

- (f) Sketch the graph of  $R(x)$ . Make sure that all of the above information is clearly visible in your graph. (It is more important that the key features are visible than for your graph to be drawn to scale.)

5. Let  $f(x) = (x + 5)(x + 3)^2(x - 171)(x - 200)^3$ .

- (a) Find the  $y$ -intercept of  $f(x)$ .
- (b) List each real zero of  $f(x)$  and its multiplicity.
- (c) For each  $x$ -intercept of  $y = f(x)$ , determine the behavior of the graph  $y = f(x)$  near that  $x$ -intercept; in particular, determine if the graph cuts the  $x$ -axis, bumps the  $x$ -axis, or slides through the  $x$ -axis.
- (d) Sketch the graph of  $y = f(x)$ , making sure that all of the above information is indicated clearly, and making sure that the end behavior of the graph is correct. (It is more important that the key features are visible than for your graph to be drawn to scale.)