Sample exam 1
Math 30, Fall 2004

This is the first exam from an old Math 30 class I taught. We have covered roughly the same material that class covered, but with a few additional topics, including the precise definition of a limit (2.4) and limits at infinity/horizontal asymptotes (2.6). Therefore, you should treat this sample exam not as a guide to what will be covered in our exam, but as a guide to what the questions will be like.

You will be allowed to use the usual calculators and **ONE** 3×5 notecard. Unless otherwise stated, you must show all your work in a problem to receive full credit.

1. (10 points) Suppose $f(x) = ab^x$ is a function whose graph has y-intercept $(0, 4)$ and passes through the point $(3, 5)$. Find the values of a and b. Show all your work.

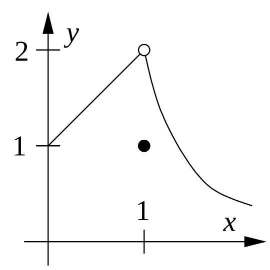
2. (12 points) Evaluate $\lim_{x \to -3} \dfrac{x^2 + 2x - 1}{7x + 2}$ as precisely as possible, if it exists, or briefly **EXPLAIN** why the limit does not exist. Show all your work.

3. (12 points) Evaluate $\lim_{x \to 2} \dfrac{x^3 + x^2 - 10}{x - 2}$ as precisely as possible, if it exists, or briefly **EXPLAIN** why the limit does not exist. Show all your work.

4. (12 points) Evaluate $\lim_{x \to 0} \dfrac{(5 + x)^2 - (5 - x)^2}{2x}$ as precisely as possible, if it exists, or briefly **EXPLAIN** why the limit does not exist. Show all your work.

5. Note that parts (a) and (b) of this problem are used in part (c).

(a) (10 points) Let $f(x)$ be a function with the following graph:



Guess the value of $\lim_{x \to 1} f(x)$ as best as possible, given only the above information, and **JUSTIFY** your answer in **ONE** phrase or sentence.

(b) (10 points) Let $h(x)$ be a function described by the following table of data:

x	0.9	0.99	0.999	1.001	1.01	1.1
$h(x)$	2.12232	2.05152	2.00375	1.99652	1.92523	1.87232

Guess the value of $\lim_{x \to 1} h(x)$ as best as possible, given only the above information, and **JUSTIFY** your answer in **ONE** phrase or sentence.

(Problem continued on next page.)

(c) (17 points) Now suppose that $g(x)$ is a function such that:

- For $0 < x < 1$ and $1 < x < 2$, $f(x) \leq g(x) \leq h(x)$, where f and h are the functions from parts (a) and (b) of this question, respectively; and
- $g(1) = 3$.

Based on these assumptions and your answers to parts (a) and (b), is g continuous at $x = 1$? Briefly **EXPLAIN** your answer. (Note that you can receive partial credit if you just explain what it means for g to be continuous at $x = 1$.)

6. (17 points) Give (i.e., describe) an example of a function $f(x)$ such that:

- $f(1.9) = 0$, $f(1.99) = 0$, $f(1.999) = 0$, $f(1.9999) = 0$, and so on; and
- $f(2.1) = 0$, $f(2.01) = 0$, $f(2.001) = 0$, $f(2.0001) = 0$, and so on;

but nevertheless, $\lim_{x \to 2} f(x)$ does not exist. (You can describe $f(x)$ either in terms of a formula, or qualitatively, in terms of its graph.) Also, **EXPLAIN** why it is that $\lim_{x \to 2} f(x)$ does not exist, even though $f(x)$ satisfies the above conditions.