

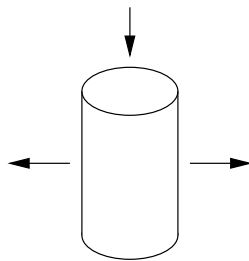
Sample exam 3
Math 30, Fall 2004

These are exam questions from an old Math 30 class. As usual, you should treat this sample exam not as a guide to what will be covered in our exam, but as a guide to what the questions will be like. (For example, we have covered hyperbolic trig functions and inverse trig functions, both of which were not covered in my old class.) As always, your best guide to what will be on the exam is the homework.

You will be allowed to use the usual calculators and **ONE** 3×5 notecard. Unless otherwise stated, you must show all your work in a problem to receive full credit.

1. Find an equation of the tangent line to the curve $y^2 = 4x^3 - 6xy$ at the point $(-2, 4)$. No explanation necessary, but show all your work.
2. An aluminum can is being crushed, as shown in the picture below. At the instant that the radius of the can is 1.5 inches, and the height of the can is 7 inches, the height of the can is decreasing by 3 inch/second, and the radius of the can is increasing by 0.25 inches/second. (In other words, the can is getting shorter and fatter.)

At that instant, is the volume of the can getting bigger or smaller? Show all your work, and briefly **EXPLAIN** your final bigger/smaller answer.



3. Suppose $g(x)$ is a function such that $g(4) = -3$, $g'(4) = 2$, and $g''(4) = -1$.
 - (a) Use the linear approximation of g at $x = 4$ to estimate the value of $g(3.97)$.
 - (b) Is your estimate of $g(3.97)$ in part (a) more likely to be greater than the actual value of $g(3.97)$ or less than the actual value of $g(3.97)$? Briefly **EXPLAIN** your answer in terms of the graph of $g(x)$ and the given data. In particular, you should sketch part of the graph of $g(x)$.
4. Suppose $f(x)$ is a function such that

$$f'(x) = xe^{2x}.$$

Note that this is the formula for the **derivative** of f , not the formula for f .

- (a) For which values of x is f increasing? **EXPLAIN** your answer in **ONE** sentence, and show all your work.
- (b) For which values of x is f concave down? **EXPLAIN** your answer in **ONE** sentence, and show all your work.

(Continued on next page.)

5. Suppose $f(x)$ is a function such that

$$f'(x) = x^2e^{-x} + 2xe^{-x}.$$

Note that this is the formula of the **DERIVATIVE** of $f(x)$, not $f(x)$ itself.

- (a) Find the critical numbers of $f(x)$, and identify each as a local minimum, local maximum, or neither.
- (b) Find the inflection points of $f(x)$, and determine the values of x for which $f(x)$ is concave up.
- (c) Use the above information to sketch the graph of $f(x)$. Make sure that all of the above features can be seen clearly in your graph.

Show all your work, and put all of your final answers in exact form (i.e., not a calculator approximation).