**Sample final exam**
**Math 30, Fall 2004**

This is part of the final from an old Math 30 class, plus a few random questions. As usual, you should treat this sample exam not as a guide to what will be covered in our exam, but as a guide to what the questions will be like. As always, your best guide to what will be on the exam is the homework.

You will be allowed to use the usual calculators and **ONE** $3 \times 5$ notecard. Unless otherwise stated, you must show all your work in a problem to receive full credit.

**1.** (a) $f(x) = e^{3x}\sin(6x^2 - 1)$. Find $f'(x)$. (b) $z = \dfrac{t^2 - 7t}{(11t + 3)^3}$. Find $\dfrac{dz}{dt}$.

**2.** Find the value of $\dfrac{dy}{dx}$ at the point $(2, -1)$ on the curve $xy^2 - \dfrac{6}{x} = y^3$.

**3.** Find the equation of the tangent line to the curve $y = \dfrac{4}{x^2} - \ln(x + 1)$ at $x = 2$.

**4.** Let $f(x) = x^2 - x$. (a) Write down the **limit definition** of $f'(2)$. (b) Using the limit laws and other properties of limits, calculate the value of $f'(2)$.

**5.** Give an example of a function $h(x)$ such that $\lim_{x\to -1} h(x) = 4$ and $h(x)$ is not continuous at $x = -1$. Describe your example either by sketching the graph of $h(x)$, by giving a formula for $h(x)$, or both. Briefly **EXPLAIN** (in a sentence or two) how you know that $h(x)$ satisfies the above conditions.

**6.** Suppose $f(x)$ is a function such that

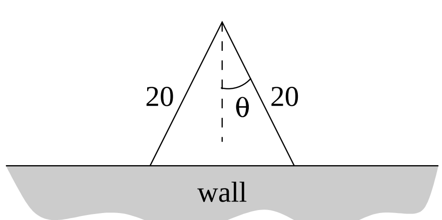
$$f'(x) = x^2e^{-x} + 2xe^{-x}.$$

(a) Find the critical numbers of $f(x)$, and identify each as a local minimum, local maximum, or neither.

(b) Find the inflection points of $f(x)$, and determine the values of $x$ for which $f(x)$ is concave up.

(c) Use the above information to sketch the graph of $f(x)$. Make sure that all of the above features can be seen clearly in your graph.

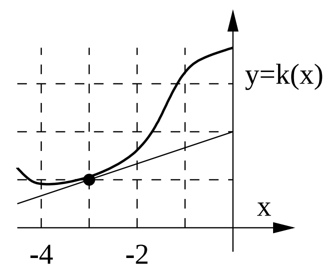
**7.** Trevor is building a stage where the heavy metal band Suiciety is to perform. The stage is to be shaped like an isoceles triangle, bounded on one side by a wall, and the equal sides of the triangle are both to be 20 feet long, as shown in the picture below. If Trevor wants the stage area to be as large as possible, what should he choose for the angle marked $\theta$ in the picture below?



**8.** Suppose $g(x)$ is a function such that $g(1) = 5$, $g'(1) = -3$, and $g''(1) = -2$. Use the tangent line approximation (linear approximation) for $g(x)$ at $x = 1$ to approximate the value of $g(0.98)$, and **EXPLAIN**, using a graph or sketch, whether your approximation is more likely to be greater than or less than the actual value of $g(0.98)$.

**9.** Let $f(x)$ and $g(x)$ be functions described by the table of data shown below, and let $k(x)$ be the function whose graph is shown below. (The graph of $k(x)$ is drawn with heavy lines; the thin straight line is the tangent line to $k(x)$ at $x = -3$.)

| $x$ | $f(x)$ | $g(x)$ | $f'(x)$ | $g'(x)$ |
|---|---|---|---|---|
| $-3$ | 1 | 3 | $-5.03$ | $-2.16$ |
| $-1$ | 3 | 0 | $-1.97$ | $-1.25$ |
| 1 | 0 | $-1$ | $-1.51$ | 1.06 |
| 3 | $-1$ | 1 | $-1.33$ | 2.02 |




(a) Let $h(x) = f(g(x))$. Compute $h'(1)$, rounding your final answer to 2 decimal places, if necessary.

(b) Let $m(x) = g(x)k(x)$. Compute $m'(-3)$, rounding your final answer to 2 decimal places, if necessary.

**10.** Let $f(x)$ be a function such that

$$f'(x) = (x - 2)e^x.$$

Note that this is the formula of the **DERIVATIVE** of $f(x)$, not $f(x)$ itself. All parts of this question refer to this same function $f(x)$, in the domain $0 \le x \le 4$.

(a) Find the exact values of all critical numbers of $f(x)$ for $0 < x < 4$, and classify each critical number as a local minimum, local maximum, or neither. Show all your work.

(b) Find the exact values of all inflection points of $f(x)$ for $0 < x < 4$. Show all your work.

(c) Now assuming that $f(0) = 3$, sketch the graph of $f(x)$ for $0 \le x \le 4$. Clearly label all critical numbers, local maxima and minima, and inflection points, and make sure that the concavity of $f(x)$ is clear throughout your graph.

(d) Find the $x$ value(s) for the **global** minimum(s) of $f(x)$ in $0 \le x \le 4$. Briefly **EXPLAIN** your answer, including how you know that your minima are global. You may use the graph of $f'(x)$ sketched below, if you find it helpful.