Sample Exam 1
Math 31, Spring 2007

1. (24 points) Compute the following.

(a) Let $g(x) = \displaystyle\int_0^x \sin(t^2)\,dt$. Calculate $g'(x)$.

(b) $\displaystyle\int \left(\frac{5}{\sqrt{x}} + 4e^{-2x}\right) dx$

(c) $\displaystyle\int x^2 \cos(5x^3 - 2)\,dx$

(d) $\displaystyle\int_0^3 \frac{x}{x^2+2}\,dx$

2. (12 points) Find the area of the region enclosed by the curves $y = 2x$ and $y = 2x^4$. No explanation necessary, but show all your work, and please do not simplify your final numerical answer.

3. (12 points) Suppose $f(x)$ and $g(x)$ are continuous functions such that

$$\int_0^4 f(x)\,dx = 3, \qquad \int_3^4 f(x)\,dx = 1, \qquad \int_0^3 g(x)\,dx = -4.$$

Calculate the value of $\displaystyle\int_0^3 (5f(x) + 7g(x))\,dx$. No explanation necessary, but show all your work, and please do not simplify your final numerical answer.

4. (16 points) Let $f(x)$ be an **INCREASING** function described by the following table.

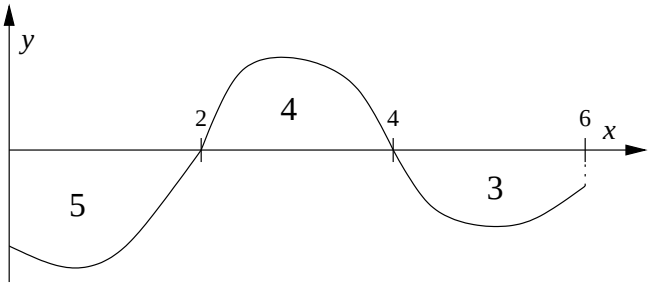
x	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00
$f(x)$	10	11	13	14	16	18	21	25	31

(a) Use four rectangles to compute the L_4 estimate (sample points are left endpoints) of the integral $\displaystyle\int_1^3 f(x)\,dx$. No explanation necessary, but show all your work.

(b) Is your answer in part (a) greater than the actual value of $\displaystyle\int_1^3 f(x)\,dx$, or less than the actual value of $\displaystyle\int_1^3 f(x)\,dx$? Briefly **explain** your answer, using the graph of $f(x)$.

5. (16 points) Find the volume of the solid obtained by rotating the region bounded by the curves $x = 0$, $x = 2$, $y = x^2 + 1$, and $y = \dfrac{x}{2}$ around the x-axis. Sketch the region, the solid, and a typical disk or washer. No explanation necessary, but show all your work, and please do not simplify your final numerical answer.

6. (20 points) Below is the graph $y = h'(x)$ of the **DERIVATIVE** $h'(x)$ of a function $h(x)$ such that $h(0) = 2$. (The graph is not drawn to scale.) Each region between the graph of $h'(x)$ and the x-axis is labelled with a number that gives the area of that region.



(a) At which value(s) of x, $0 < x < 6$, does $h(x)$ have a local minimum? No explanation necessary, but show all your work.

(b) What is the **minimum** value that $h(x)$ takes for $0 \leq x \leq 6$? (I.e., what is the minimum possible $y = h(x)$ for $0 \leq x \leq 6$?) Briefly **justify** your answer using the Fundamental Theorem of Calculus, part 2.