

Sample Exam 2
Math 31, Spring 2007

1. (18 points) Compute the following.

(a) $\int x \sin 3x \, dx$

(b) $\int \frac{e^{2x}}{1 + 3e^{2x}} \, dx$

(c) $\int_3^5 \frac{x^2}{\sqrt{x^3 - 1}} \, dx$

2. (12 points) Determine if the improper integral

$$\int_1^{\infty} x e^{-x^2} \, dx$$

is convergent or divergent, and if it is convergent, find its value (evaluate the integral).

3. (12 points) Compute the following integral. **DO NOT SIMPLIFY** your final answer.

$$\int (\cos^2 2\theta)(\sin^3 2\theta) \, d\theta.$$

4. (14 points) Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the curves

$$y = \frac{1}{x^2 + 1}, \quad x = 0, \quad x = 2,$$

about the y -axis. Sketch the region and a typical shell.

5. (14 points) Compute the following integral. **DO NOT SIMPLIFY** your final answer.

$$\int \frac{x + 1}{(x - 2)(x + 5)} \, dx$$

6. (14 points) Let $f(x)$ be a function described by the following table.

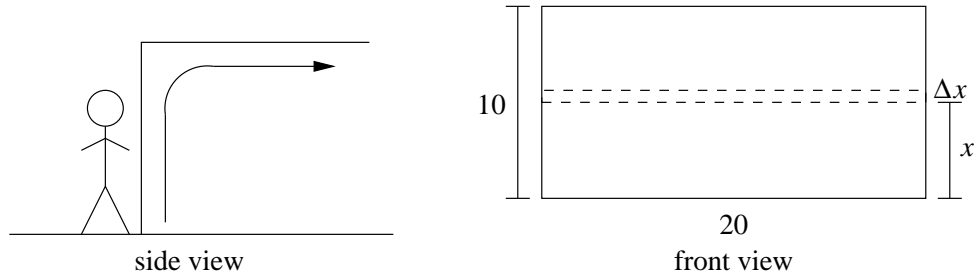
x	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00
$f(x)$	7.6	7.8	8.1	8.4	8.8	9.2	9.7	10.2	10.8

(a) Find the approximation T_4 (Trapezoidal Rule, 4 subintervals) for $\int_3^5 f(x) \, dx$. No explanation necessary, but show all your work.

(b) Find the approximation M_4 (Midpoint Rule, 4 subintervals) for $\int_3^5 f(x) \, dx$. No explanation necessary, but show all your work.

(c) Find the approximation S_8 (Simpson's Rule, 8 subintervals) for $\int_3^5 f(x) dx$. No explanation necessary, but show all your work.

7. (16 points) Penelope has a 20 foot wide by 10 foot tall garage door that she wants to raise to a uniform height of 10 feet, as shown in the side view picture below.



Suppose the garage door weighs 3 pounds per square foot of surface area.

- Suppose we consider a small slice of the garage door, of height Δx , that is x feet above the floor. (See front view picture, above.) How much work does it take to raise that small slice to a height of 10 feet? Show all your work. (If this part of the question doesn't make sense to you, go on to part (b).)
- How much work does Penelope have to do to raise all of the garage door to a height of 10 feet? (Note that if you can answer this part of the question without answering part (a), you can still receive full credit.)