

Sample Exam 3
Math 31, Spring 2007

Note that our exam will also cover section 8.3, which this exam did not cover.

1. Determine if the improper integral $\int_1^{\infty} xe^{-x^2} dx$ is convergent or divergent, and if it is convergent, find its value (evaluate the integral).

2. Let $\{b_n\}$ be the sequence $\left\{ \frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{4}, \frac{\sqrt{4}}{8}, \frac{\sqrt{5}}{16}, \dots \right\}$. Find a formula for the n th term b_n

of this sequence, assuming the sequence begins with $b_1 = \frac{\sqrt{2}}{2}$.

3. Let $\{a_n\}$ be the sequence given by $a_n = \frac{n^2 + 1}{n!}$.

(a) Write down the first four terms of the sequence $\{a_n\}$, i.e., write down the values of a_1, a_2, a_3, a_4 .

(b) Determine the value of

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^2 + 1}{n!},$$

if the limit exists; or if the limit does not exist, **explain** how you know the limit does not exist. Briefly **justify** your answer.

4. Determine if the series

$$\sum_{n=0}^{\infty} \frac{7(3^n)}{5^n}$$

converges or diverges. If it converges, find its sum. Briefly **JUSTIFY** your answer.

5. Use the integral test to determine if the series

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$$

converges or diverges. Briefly **JUSTIFY** your answer.

6. Determine if the series

$$\sum_{n=1}^{\infty} \frac{n^3 + 1}{2n^4 - n}$$

converges or diverges. Briefly **JUSTIFY** your answer.

7. Determine if the series

$$\sum_{n=1}^{\infty} \frac{\sqrt{2n-1}}{n^3}$$

converges or diverges. Briefly **JUSTIFY** your answer.