Sample Final Exam
Math 31, Spring 2007

1. Compute the following integrals. Show all your work, and do not simplify your final answers.

(a) (8 points) $\displaystyle\int_0^2 \frac{x^6}{x^7+2}\,dx$

(b) (8 points) $\displaystyle\int xe^{2x}\,dx$

(c) (8 points) $\displaystyle\int \sin^3(5x)\,dx$

2. (10 points) Let $z = 3+2i$ and $w = 1-7i$. Calculate $\dfrac{z}{w}$, $\overline{z}$, and $|z|$ (the modulus of z). No explanation necessary, but show all your work.

3. (10 points) Let $z = -\sqrt{3}+i$. Calculate the polar form of z (i.e., calculate r and θ such that $z = re^{i\theta}$), and use that polar form to calculate the polar form of z^{10}. No explanation necessary, but show all your work.

4. (14 points) Determine if the series

$$\sum_{n=1}^{\infty} \frac{7n^2}{2n^4+n+1}$$

converges or diverges. Briefly **JUSTIFY** your answer.

5. (14 points) Determine if the series

$$\sum_{n=1}^{\infty} \frac{\sqrt{2n+7}}{3n}$$

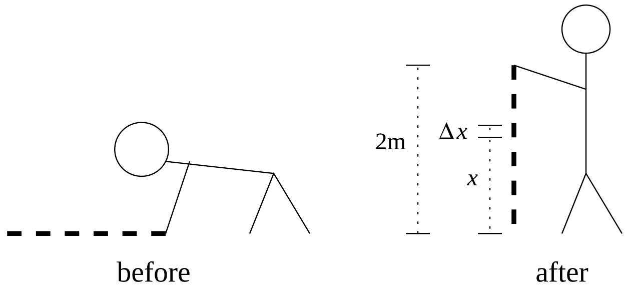
converges or diverges. Briefly **JUSTIFY** your answer.

6. (16 points) At time $t = 0$, Enrique suddenly realizes that he is 4 feet away from a stink bomb about to go off. He immediately runs directly away from the bomb at a velocity of $v(t) = \sqrt{7t+1}$ feet per second until the bomb goes off at time $t = 10$. How far away is Enrique from the stink bomb when it goes off at time $t = 10$?

No explanation necessary, but show all your work. You do not need to simplify your final numerical answer.

7. (16 points) Find the volume of the solid obtained by rotating the region bounded by the curves $x = 5$, $x = 7$, $y = 0$, and $y = 1 + x^3$ around the x-axis. Sketch the region, the solid, and a typical disk or washer. No explanation necessary, but show all your work, and please do not simplify your final numerical answer.

8. (18 points) Nima grabs a 2 meter long chain lying on the floor, with a mass of 3 kg/m, and lifts one end of the chain to a height of 2 meters, lifting the rest of the chain along with it, as shown in the picture below.



(a) Suppose we consider a small piece of the chain of length Δx meters (see "after" picture, above). What is the mass of this piece of chain?

(b) Suppose we consider a small piece of the chain of length Δx meters that Nima lifts to a height of x meters (see "after" picture, above). How much work did Nima have to do to lift that piece of chain of length Δx meters to a height of x meters? Note that acceleration due to gravity is 9.8 m/sec^2. (If this part of the question doesn't make sense to you, go on to part (c).)

(c) How much total work does Nima have to do to lift one end of the chain to a height of 2 meters? No explanation necessary, but show all your work. Note that if you can answer this part of the question without answering part (b), you can still receive full credit.

9. (18 points) Consider the function $f(x) = e^{3x}$.

1. Fill in the missing entries in the table below.
2. Use the pattern you see in the table to find a formula for the Maclaurin series of $f(x)$ (i.e., the Taylor series of $f(x)$ centered at 0). You do not need to calculate the radius of convergence of the Maclaurin series.

$f(x) = e^{3x}$	$f(0) =$	$c_0 =$
$f'(x) = 3e^{3x}$	$f'(0) =$	$c_1 =$
$f''(x) = 3^2e^{3x}$	$f''(0) =$	$c_2 =$
$f'''(x) =$	$f'''(0) =$	$c_3 =$
$f^{(4)}(x) =$	$f^{(4)}(0) =$	$c_4 =$

10. (18 points) Consider the power series

$$\sum_{n=0}^{\infty} \frac{7x^n}{n^2 4^n}.$$

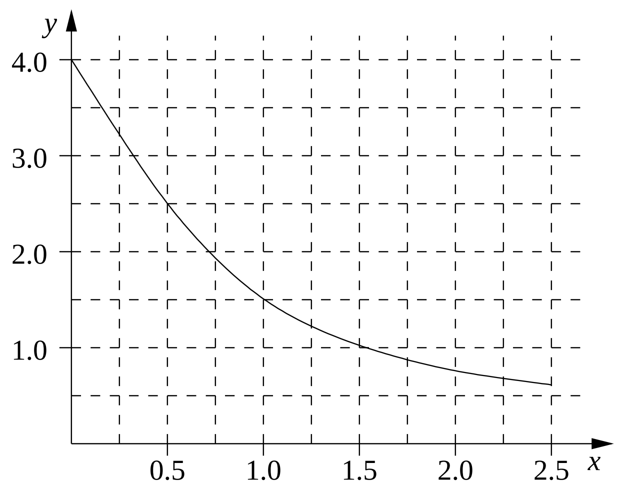
(a) Find the radius of convergence of the above power series. (You do not need to worry about endpoints or find the interval of convergence.)

(b) Consider the following two series.

$$(1) \qquad \sum_{n=0}^{\infty} \frac{7(0.1^n)}{n^2 4^n} \qquad\qquad (2) \qquad \sum_{n=0}^{\infty} \frac{7((-35)^n)}{n^2 4^n}$$

Do both series (1) and series (2) converge? Do they both diverge? Does one converge and the other diverge? Briefly **explain** your answer, **without further calculation**, based only on your answer in part (a) of this problem.

11. (20 points) Consider the function $y = f(x)$ whose graph is shown in the picture below.



(a) Find the approximations L_5 (Left endpoint Rule, 5 subintervals) and R_5 (Right endpoint Rule, 5 subintervals) for the area between $y = 0$, $y = f(x)$, $x = 0$, and $x = 2.5$. Show all your work.

(b) Find the approximation T_5 (Trapezoidal Rule, 5 subintervals) for the area between $y = 0$, $y = f(x)$, $x = 0$, and $x = 2.5$. Show all your work.

(c) Is L_5 greater than or less than the actual area between $y = 0$, $y = f(x)$, $x = 0$, and $x = 2.5$? Is T_5 greater than or less than the actual area between $y = 0$, $y = f(x)$, $x = 0$, and $x = 2.5$? Briefly **explain** your answers with a picture.

12. (22 points)

(a) What is the Maclaurin series (Taylor series with center $x = 0$) of $\cos x$? No explanation necessary.

(b) Use the series you wrote down in part (a) to find the Maclaurin series of $\cos x^2$. Show all your work.

(c) Use the first three terms of the Maclaurin series of $\cos x^2$ to approximate the following integral:

$$\int_0^{0.6} \cos x^2 \, dx.$$

Show all your work.