

Sample Exam 3, Math 32, Fall 2006

The following questions are included only to indicate the style of questions that will appear on Exam 3.

1. Calculate

$$\int_3^4 \int_{-1}^2 (xy^2 - x^3) dx dy.$$

No explanation necessary, but show all your work. Do not simplify your final numerical answer.

2. Let $f(x, y)$ be a differentiable function, and suppose that f is described by the following table.

(x, y)	f	f_x	f_y	f_{xx}	f_{xy}	f_{yy}
$(1, -4)$	0	0	1	0	1	1
$(3, 5)$	-3	0	0	2	-2	3
$(-2, 7)$	1	4	1	1	0	2
$(4, 1)$	6	0	0	1	-3	2
$(0, -5)$	5	0	0	-1	1	-2

Suppose also that $f_x(x, y) \neq 0$ for all (x, y) not listed in the table.

- List all of the critical points of f and briefly **EXPLAIN** why these are the **only** possible critical points of f .
- Classify the critical points you found in part (a) (as local maximum, etc.). Show all your work.

3. Consider the double integral

$$\int_{-1}^1 \int_{x^2}^1 f(x, y) dy dx.$$

- Draw the region of integration.
- Change the order of integration, i.e., express this integral as an iterated integral, integrating the variable x first. **DO NOT EVALUATE THIS INTEGRAL.** Also, make sure that the limits of integration are clearly readable; unclear answers will not receive full credit.

4. Let D be a flat washer-shaped plate in the xy -plane that is bounded by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$, and suppose that the density of D at (x, y) is $\rho(x, y) = x^2 + 2$. Find the mass of the flat plate D . No explanation necessary, but show all your work. **DO NOT SIMPLIFY** your final numerical answer.

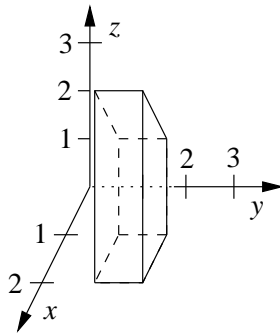
5. Let $f(x, y)$ be a function such that

$$f_x(x, y) = x - y, \quad f_y(x, y) = y^2 - x - 2.$$

(You do not need to find a formula for $f(x, y)$; in fact, having such a formula is not particularly helpful in solving this problem.)

- (a) Find all critical points of $f(x, y)$. (Hint: There are two critical points, and one of them is $(2, 2)$.) Show all your work.
- (b) Classify each critical point of $f(x, y)$ as a local minimum, local maximum, or saddle point. Show all your work.

6. Let E be the three-dimensional region described by $1 \leq x \leq 2$, $2 \leq y \leq 3$, $0 \leq z \leq 2x$, as shown in the following picture.



Calculate the triple integral $\iiint_E y^2 \sin(x^2 + 1) dV$. Show all your work, and **DO NOT SIMPLIFY** your final numerical answer.