

1. Let  $a_1, b_1, a_2, b_2 > 0$  be given and

$$a'_2 + b'_1 = a_2 + b_2, \quad a'_1 b'_1 = a_2 b_1, \quad a'_1 = a_1 + b_1, \quad a'_2 b'_2 = b_2.$$

Write down expressions for computing  $a'_1, a'_2, b'_1, b'_2$  that involve no subtractions as well as the sequence in which the operations need to be performed.

2. Prove or disprove: If  $A$  is symmetric positive definite, then  $A + I$  is symmetric positive definite.
3. Prove that if  $\|A\| < 1$ , then  $A - I$  is nonsingular. ( $\|\cdot\|$  stands for any operator norm).
4. If  $A$  is a projector matrix, i.e.,  $A^2 = A$ , prove that  $\|A\|_2 \geq 1$ .
5. Prove that if  $A$  is orthogonal, then  $\kappa_2(A) = 1$ .
6. If  $A = U\Sigma V^T$  is the SVD of the square matrix  $A$ , find the SVD of  $A^T A$  and  $AA^T$ .
7. Find the condition number of the matrix

$$A = \begin{pmatrix} 0 & 2 \\ 2 & 3 \end{pmatrix}$$

in the 1, 2, and  $\infty$  norms.