1. Let \( a_1, b_1, a_2, b_2 > 0 \) be given and

\[
\begin{align*}
a'_2 + b'_1 &= a_2 + b_2, \\
a'_1 b'_1 &= a_2 b_1, \\
a'_1 &= a_1 + b_1, \\
a'_2 b'_2 &= b_2.
\end{align*}
\]

Write down expressions for computing \( a'_1, a'_2, b'_1, b'_2 \) that involve no subtractions as well as the sequence in which the operations need to be performed.

2. Prove or disprove: If \( A \) is symmetric positive definite, then \( A + I \) is symmetric positive definite.

3. Prove that if \( \|A\| < 1 \), then \( A - I \) is nonsingular. (\( \| \cdot \| \) stands for any operator norm).

4. If \( A \) is a projector matrix, i.e., \( A^2 = A \), prove that \( \|A\|_2 \geq 1 \).

5. Prove that if \( A \) is orthogonal, then \( \kappa_2(A) = 1 \).

6. If \( A = U\Sigma V^T \) is the SVD of the square matrix \( A \), find the SVD of \( A^T A \) and \( AA^T \).

7. Find the condition number of the matrix

\[
A = \begin{pmatrix} 0 & 2 \\ 2 & 3 \end{pmatrix}
\]

in the 1, 2, and \( \infty \) norms.