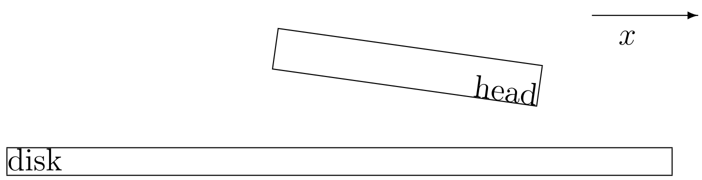
Math 243A Final Project

A problem of great interest in the data storage industry is the sliding of a reader over the surface of a disk drive. In the sketch below the head flies from right to left.



The flight elevation $h(x)$ decreases from the front of the slider to the back. At the front the elevation is 0.38×10^{-6} meters; at the back, it is 0.127×10^{-6} meters. In steady state, the pressure distribution under the slider is given by the Reynolds equation:

$$\frac{\partial}{\partial x}\left(h^3 P \frac{\partial P}{\partial x}\right) + \frac{\partial}{\partial y}\left(h^3 P \frac{\partial P}{\partial y}\right) = 6\mu U \frac{\partial (Ph)}{\partial x}$$

The value of μ is 1.81×10^{-6} Pa-sec; the value of U is 50.80m/sec.; the head is 2.54×10^{-6} meters in the X direction, and 2.54×10^{-6} meters in the Y direction. The pressure around the edge of the head is $101,325$ Pa.

One can discretize the left hand side using

$$\begin{aligned}\frac{\partial}{\partial x}\left(h^3 P \frac{\partial P}{\partial x}\right) \quad &\approx \quad \frac{1}{\Delta x}\left[\left(\frac{h_{i+1,j}+h_{i,j}}{2}\right)^3 \left(\frac{P_{i+1,j}+P_{i,j}}{2}\right)\left(\frac{P_{i+1,j}-P_{i,j}}{\Delta x}\right)\right. \\ &\left. - \left(\frac{h_{i,j}+h_{i-1,j}}{2}\right)^3 \left(\frac{P_{i,j}+P_{i-1,j}}{2}\right)\left(\frac{P_{i,j}-P_{i-1,j}}{\Delta x}\right)\right]\end{aligned}$$

for a uniform Δx and uniform Δy grid. Doing the same for the other terms of the equation, solve for the pressure under the head using Gauss/Seidel or SOR.

Turn in:

1. The finite difference scheme and its derivation
2. The computer program
3. A legible plot of the steady-state pressure distribution
4. The highest pressure on the head (computed with at least 4 accurate digits).