

Math 243A, Homework # 3, Due 9/27/2012

1. Solve $u_t = u_{xx}$ on $-1 \leq x \leq 1$ with initial data

$$u_0(x) = \begin{cases} 1 & \text{if } |x| < \frac{1}{2} \\ \frac{1}{2} & \text{if } |x| = \frac{1}{2} \\ 0 & \text{if } |x| > \frac{1}{2}. \end{cases}$$

Solve up to $t = \frac{1}{2}$. The boundary data and the exact solutions are given by

$$u(t, x) = \frac{1}{2} + 2 \sum_{l=0}^{\infty} (-1)^l \frac{\cos \pi(2l+1)x}{\pi(2l+1)} e^{-\pi^2(2l+1)^2 t}.$$

Use the Crank–Nicolson scheme

$$\frac{v_m^{n+1} - v_m^n}{k} = \frac{1}{2} \left(\frac{v_{m+1}^{n+1} - 2v_m^{n+1} + v_{m-1}^{n+1}}{h^2} + \frac{v_{m+1}^n - 2v_m^n + v_{m-1}^n}{h^2} \right)$$

with $h = \frac{1}{10}, \frac{1}{20}, \frac{1}{40}$ and $k = h$. Compute the 2-norm of the error in each case.