

Math 243A, Homework # 4, Due 10/4/2012

1. Prove that the $(N - 1) \times (N - 1)$ tridiagonal matrix

$$A = \begin{bmatrix} 0 & b & & & & \\ c & 0 & b & & & \\ & c & 0 & b & & \\ & & \ddots & \ddots & \ddots & \\ & & & \ddots & \ddots & b \\ & & & & c & 0 \end{bmatrix}$$

has eigenvalues $\lambda_s = 2\sqrt{bc} \cos \frac{s\pi}{N}$, $s = 1, 2, \dots, N - 1$ and eigenvectors v_s whose j th component is

$$(c/b)^{j/2} \sin \frac{js\pi}{N}.$$