1. $f(x,y,z) = xe^y + ye^z + ze^x$ at $(0,0,0)$ in the dir. of $v = \langle 5,1,2\rangle$

First we compute $u = \frac{1}{|v|}\cdot v$; u will be of length 1.

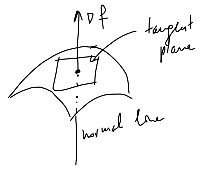
$$u = \frac{1}{\sqrt{5^2+1^2+2^2}}\langle 5,1,2\rangle = \frac{1}{\sqrt{30}}\langle 5,1,2\rangle = \left\langle \frac{5}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{2}{\sqrt{30}}\right\rangle$$

$$\boxed{D_u f = f_x\cdot\frac{5}{\sqrt{30}} + f_y\cdot\frac{1}{\sqrt{30}} + f_z\cdot\frac{2}{\sqrt{30}}} = (e^y + ze^x)\frac{5}{\sqrt{30}} + (xe^y + e^z)\cdot\frac{1}{\sqrt{30}} + (ye^z + e^x)\frac{2}{\sqrt{30}}$$

$$D_u f(0,0,0) = (e^0 + 0\cdot e^0)\frac{5}{\sqrt{30}} + (0\cdot e^0 + e^0)\frac{1}{\sqrt{30}} + (0\cdot e^0 + e^0)\frac{2}{\sqrt{30}} = \frac{5}{\sqrt{30}} + \frac{1}{\sqrt{30}} + \frac{2}{\sqrt{30}} = \boxed{\frac{8}{\sqrt{30}}}$$

Variations: 1) Different f ;
2) The variables can have different names, e.g., $f(u,v,w) = ue^v + ve^w + we^u$
3) The vector can be given as "from point $A(0,1,2)$ to point $B(-1,3,2)$".
The vector is then $\vec{AB} = \langle -1-0,\ 3-1,\ 2-2\rangle$.

2. Tangent plane and normal line to $x^2 + 2y^2 + 3z^2 = 6$ at $(1,1,1)$.



The surface is defined as $f(x,y,z) = 6$, where

$$f(x,y,z) = x^2 + 2y^2 + 3z^2$$

$$\boxed{\nabla f} = \langle f_x, f_y, f_z\rangle = \langle 2x, 4y, 6z\rangle$$

$$\nabla f(1,1,1) = \langle 2\cdot 1, 4\cdot 1, 6\cdot 1\rangle = \langle 2,4,6\rangle.$$

The tangent plane is $2(x-1) + 4(y-1) + 6(z-1) = 0$.

The normal line is
$$\begin{cases} x = 1 + t\cdot 2 \\ y = 1 + t\cdot 4 \\ z = 1 + t\cdot 6 \end{cases}$$

3. $\frac{\partial M}{\partial u}$ and $\frac{\partial M}{\partial v}$ for $M = xe^{y-z^2}$, $x = 2uv$, $y = u - v$, $z = u + v$

chain Rule:
$$\frac{\partial M}{\partial u} = \frac{\partial M}{\partial x}\cdot\frac{\partial x}{\partial u} + \frac{\partial M}{\partial y}\cdot\frac{\partial y}{\partial u} + \boxed{\frac{\partial M}{\partial z}\cdot\frac{\partial z}{\partial u}}$$
$$= e^{y-z^2}\cdot 2v + xe^{y-z^2}\cdot 1 + xe^{y-z^2}(-2z)\cdot 1$$
$$= e^{y-z^2}(2v + x - 2xz) \quad \text{plug } u \text{ and } v \text{ back in}$$
$$= e^{u-v-(u+v)^2}(2v + 2uv - 2\cdot 2uv(u+v)).$$

$$\frac{\partial M}{\partial v} = \frac{\partial M}{\partial x}\cdot\frac{\partial x}{\partial v} + \frac{\partial M}{\partial y}\cdot\frac{\partial y}{\partial v} + \frac{\partial M}{\partial z}\cdot\frac{\partial z}{\partial v}$$
$$= e^{y-z^2}\cdot 2u + \cancel{xe^{y-z^2}}\cdot(-1) + \cancel{xe^{y-z^2}(-2z)}\cdot 1$$
$$= e^{y-z^2}(2u - x - 2xz)$$
$$= e^{u-v-(u+v)^2}(2u - 2uv - 2\cdot 2uv\cdot(u+v)).$$

Midterm 2 covers chapters 14.1 - 14.6.

4. $f(x,y,z) = x\sqrt{y-z}$ $\quad f_{xyz} = ?$ $\quad ((f_x)_y)_z = \boxed{f_{xyz}}$

$$f_x = \sqrt{y-z}$$
$$f_{xy} = (f_x)_y = \left((y-z)^{\frac{1}{2}}\right)_y = \frac{1}{2}(y-z)^{-\frac{1}{2}}\cdot 1 = \frac{1}{2}(y-z)^{-\frac{1}{2}}$$
$$f_{xyz} = (f_{xy})_z = \frac{1}{2}(y-z)^{-\frac{3}{2}}\left(-\frac{1}{2}\right)(-1) = \frac{1}{4}(y-z)^{-\frac{3}{2}}.$$

Expect: 1. Any partial derivatives of any functions.
2. Explicit or implicit

5. Implicit derivatives:

$yz + x\ln y = z^2$ $\quad$ find $\frac{\partial z}{\partial y} = ?$

(z is defined implicitly as a function of x and y).

$$\frac{\partial}{\partial y}(y\cdot z + x\ln y) = \frac{\partial}{\partial y}(z^2)$$

x is constant
y is y
z is a function of x and y

$$1\cdot z + y\cdot\frac{\partial z}{\partial y} + x\cdot\frac{1}{y} = 2z\cdot\frac{\partial z}{\partial y}$$

Now we solve for $\frac{\partial z}{\partial y}$:

$$y\frac{\partial z}{\partial y} - 2z\frac{\partial z}{\partial y} = -z - \frac{x}{y} \quad\Rightarrow\quad \frac{\partial z}{\partial y}(y - 2z) = -z - \frac{x}{y}$$
$$\Rightarrow\quad \frac{\partial z}{\partial y} = \frac{-z - \frac{x}{y}}{y - 2z}.$$