San Jose State University
Department of Mathematics

Math 32, Instructor: Plamen Koev, Practice Final Exam, Fall 2008

Formulas you can use:

$$L = \int_a^b |r'(t)|dt, \quad T(t) = \frac{r'(t)}{|r'(t)|}, \quad N(t) = \frac{T'(t)}{|T'(t)|}, \quad B(t) = T(t) \times N(t), \quad \kappa(t) = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3}.$$

$$v(t) = r'(t), \quad a(t) = v'(t).$$

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1. Consider the curve $r(t) = \langle \sin^2(t) - 1, \cos^2(t) + 1 \rangle$.

   (a) Find its curvature at $t = \pi/2$.

2. Find the volume of the parallelepiped defined by the vectors $a = \langle 1, 1, 1 \rangle$, $b = \langle 1, -1, 1 \rangle$, and $c = \langle -1, 1, 1 \rangle$.

3. Find the angle between the planes $x + 2y - 2z = 1$ and $2x - y - z = 6$.

4. Find the angle of intersection of the curves $r_1(t) = \langle t + 1, 1 - t, 3 + t^2 \rangle$ and $r_2(s) = \langle 4 - s, s - 2, s^2 \rangle$.

5. Find the length of the curve $r(t) = \langle 1, t^2, t^2 \rangle$ for $0 \leq t \leq 1$.

6. Consider the curve $r(t) = \langle \cos t + 1, \sin t, \ln \cos t \rangle$.

   - Find its unit tangent, unit normal, and binormal vectors at the point $(2, 0, 0)$.
   - Find the equations of its normal and osculating planes at the point $(2, 0, 0)$.

7. Find the extreme values of $f(x, y) = e^{xy}$ on $x^3 + y^3 \leq 16$.

8. Find the derivative of the function $f(x, y, z) = x + xy^2$ at the point $(0, 0, 0)$ in the direction of the vector $\langle 1, 2, 2 \rangle$.

9. Evaluate the integral:

$$\iint_R xy\sqrt{x^2 + y^2}\, dA, \quad R = [0, 1] \times [0, 1].$$

10. Evaluate the integral:

$$\iint_D y^3\, dA, \text{ where } D \text{ is the triangular region with vertices } (0, 2),\ (1, 1),\ (3, 2).$$

11. Evaluate the integral by reversing the order of integration

$$\int_0^1 \int_{3y}^3 e^{x^2}\, dx\, dy.$$

12. Use polar coordinates to find the volume of the solid bounded by the paraboloid $z = 1 + 2x^2 + 2y^2$ and the plane $z = 7$ in the first quadrant.

13. Evaluate the integral by changing to cylindrical coordinates:

$$\iiint_B xz \, dA, \quad A = \{(x, y, z) \mid 0 \le y \le 2, \ -\sqrt{4 - y^2} \le x \le \sqrt{4 - y^2}, \ \sqrt{x^2 + y^2} \le z \le 2\}.$$

Final:

$$\iiint_B \sqrt{x^2 + y^2} \, dA, \quad A = \{(x, y, z) \mid -3 \le x \le 3, \ 0 \le y \le \sqrt{9 - x^2}, \ 0 \le z \le 9 - x^2 - y^2\}$$

14. Using spherical coordinates evaluate $\iiint_E z \, dV$, where E is enclosed by the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$ in the first octant.

15. Final: Using spherical coordinates evaluate $\iiint_E e^{\sqrt{x^2+y^2+z^2}} \, dV$, where E is enclosed by the sphere $x^2 + y^2 + z^2 = 9$ in the first octant.