Some Problems on Elementary Matrices

Exercise 1. Find a matrix $E_1$ such that if $B$ is a $3 \times 10$ matrix, then $E_1B$ is $B$ with its third row replaced by four times its first row added to its third row.

Answer 1. Apply the given row operation—replace the third row by adding four times the first row to the third row—to $I_3$, the $3 \times 3$ identity matrix to get $E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix}$.

Exercise 2. Suppose $E_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. If $C$ is a $4 \times 500$ matrix, describe in words the matrix $E_2C$.

Answer 2. $E_2C$ is the matrix $C$ with its second and third rows exchanged.

Exercise 3. Let $G = \begin{bmatrix} 2 & 3 & 6 & 4 \\ 4 & 7 & 2 & 5 \\ 0 & 3 & 8 & 7 \end{bmatrix}$ and $H = \begin{bmatrix} 2 & 3 & 6 & 4 \\ 0 & 1 & -10 & -3 \\ 0 & 3 & 8 & 7 \end{bmatrix}$ find $E_3$ such that $H = E_3G$.

Answer 3. Since $H$ is $G$ with its second row replaced by $-2$ times its first row added to its second row, we perform that same elementary row operation on $I_3$ to get $E_3 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Exercise 4. Let $K = \begin{bmatrix} 3 & 6 \\ 8 & 24 \\ 2 & 7 \\ 0 & 9 \end{bmatrix}$ and $L = \begin{bmatrix} 3 & 6 \\ 1 & 3 \\ 2 & 7 \\ 0 & 9 \end{bmatrix}$ find $E_4$ such that $L = E_4K$.

Answer 4. Since $L$ is $K$ with its second row scaled by a factor of $1/8$, we perform that same elementary row operation on $I_4$ to get $E_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/8 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.