The Invertible Matrix Theorem

**Theorem.** Let $A$ be a square $n \times n$ matrix. Then the following statements are equivalent. That is, for a given $A$, the statements are either all true or all false.

a. $A$ is an invertible matrix.

b. $A$ is row equivalent to the $n \times n$ identity matrix, $I_n$.

c. $A$ has $n$ pivot positions.

d. The equation $Ax = 0$ has only the trivial solution.

e. The columns of $A$ form a linearly independent set in $\mathbb{R}^n$.

f. The linear transformation given by $x \mapsto Ax$ is one-to-one.

g. The equation $Ax = b$ has at least one solution for each $b$ in $\mathbb{R}^n$.

h. The columns of $A$ span $\mathbb{R}^n$.

i. The linear transformation given by $x \mapsto Ax$ maps $\mathbb{R}^n$ onto $\mathbb{R}^n$.

j. There is an $n \times n$ matrix $C$ such that $CA = I_n$.

k. There is an $n \times n$ matrix $D$ such that $AD = I_n$.

l. $A^T$ is an invertible matrix.

m. $|A| = \det A \neq 0$

n. 0 is not an eigenvalue for $A$. 

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