Math 112 Sample Mid Term 2

All solutions must be supported by working

1. Let $z = f(x, y)$, where f has continuous partial derivatives. If we make the standard polar/rectangular substitution $x = r\cos\theta,\ y = r\sin\theta$, show that

$$(\frac{\partial z}{\partial x})^2 + (\frac{\partial z}{\partial y})^2 = (\frac{\partial z}{\partial r})^2 + \frac{1}{r^2}(\frac{\partial z}{\partial \theta})^2$$

2. Give a set of parametric equations for the normal line to the surface defined by the equation $e^{xy} + e^{xz} - 2e^{yz} = 0$ at the point $(-1,-1,-1)$.

3. Determine the moving frame $[\vec{T}, \vec{N}, \vec{B}]$ for the path $\vec{x}(t) = (e^{2t}\sin t, e^{2t}\cos t, 1)$.

4. Show that (a) $div(\varphi\ grad\ \varphi) = \|grad\ \varphi\|^2 + \varphi\nabla^2\varphi$, (b) $curl(r^n\vec{r}) = 0$

5. Find first and second order Taylor polynomials for the function $f(x, y) = e^{2x}\cos(3y)$ at the point $(0, \pi)$.

6. Identify and determine the nature of the critical points of $f(x, y) = 2x - 3y + \ln(xy)$.