(a) Complete the truth table for \((\neg p \lor q) \lor (p \land \neg q)\):

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>\neg p</th>
<th>\neg q</th>
<th>(\neg p \lor q)</th>
<th>(p \land \neg q)</th>
<th>(\neg p \lor q) \lor (p \land \neg q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
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<td>F</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td></td>
</tr>
</tbody>
</table>

(b) Using DeMorgan’s laws, distributive laws, etc, simplify the expression as best as you can:

\[\sim((\sim p \land q) \lor (\sim p \land \sim q)) \lor p\]. Write your answer here: \[\sim p\]

(c) Rewrite the statement, \((p \rightarrow q) \iff (q \rightarrow r)\), using only two symbols, \(\sim\) & \(\land\).

\[
\sim(p \rightarrow q) \land \sim(q \rightarrow r) = (\sim p \land \sim q) \land (q \land \sim r)
\]

#2 Convert the binary number, \((1101.0101)_2\), to the usual decimal system:

\[
\frac{1}{2^3} + \frac{1}{2^2} + \frac{1}{2^1} + \frac{1}{2^0} = 2.3125
\]

#3 Write negations for the following:

(a) For real numbers \(x\), if \(x > 3\) then \(x^2 > 9\).

(b) The sum of any two rational numbers is rational.

(c) There is a prime number which is even.

(d) \(0 \geq x \geq -5\), \(0 \geq x \geq 5\), and \(x > 5\).

(e) If \(n\) is odd and \(m\) is even then \(m + n\) is odd.
Rewrite in if-then form (do not use symbols and be as simple as possible):
(a) Having a large income is not a necessary condition for a person to be happy.

(b) Happiness is a sufficient condition for prosperity.

For the following arguments, check whether valid/invalid, and by which rule of inference (modus ponens, ... disjunctive syllogism, etc):
(a) If my glasses are on the kitchen table, then I saw my glasses during breakfast.
   I did not see my glasses during breakfast.
   Therefore: My glasses are not on the kitchen table.

   **Part (a): Valid or invalid?** VALID
   **If valid, which form of argument was used?** MODUS TOLLENS

(b) If my glasses are on the kitchen table, then I saw my glasses during breakfast.
   I saw my glasses during breakfast.
   Therefore: My glasses are on the kitchen table.

   **Part (b): Valid or invalid?** INVALID

(c) \( p \rightarrow q \)
   \( p \rightarrow r \)

   Therefore: \( p \rightarrow (q \land r) \).

   **Part (c): Valid or invalid?** VALID
   Prove this by constructing a truth table.

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>r</th>
<th>( p \rightarrow q )</th>
<th>( p \rightarrow r )</th>
<th>( q \land r )</th>
<th>( p \rightarrow (q \land r) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
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</tr>
</tbody>
</table>

   **Correct table "valid"**

   **Wrong titles for a column**

   **Critical rows have a true conclusion**

   **Valid**

Write a Boolean expression (but do not simplify) based on the truth table:

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>?</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
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<tr>
<td>F</td>
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<td>T</td>
</tr>
</tbody>
</table>

\( \sim (p \land q) \) or \( p \rightarrow \sim q \)

Either one is ok.

Example from page 51.
7. (15 points) Consider an empty bowl and the statement: "All the balls in the bowl are blue". Answer these:

(a) Write this statement using the symbols. ∀, ∃, p, q, r etc.

\[ \forall x \in \text{Bowl}, \ x \text{ is blue} \]

(b) Write a negation for part (a), using symbols.

\[ \exists x \in \text{Bowl}, \text{ such that, } \neg p(x) \]

(c) Write the answer to part (b) in words.

There exists a ball in the bowl, such that, the ball is not blue.

(d) Is the statement in part (c) true or false?

[FALSE]

(e) Is the original statement ".... all ball are blue", true or false?

[TRUE] because the negation, part (d) is false.

8. (12 points) Consider the statement: \( \forall x \in \mathbb{R}, \exists \) a real number y, such that, \( x - y = 7 \). Answer these:

(a) Rewrite the statement in English, without using symbols. Recall that \( \mathbb{R} \) is the set of real numbers.

For all real numbers \( x \), there exists a real number \( y \), such that, the difference of \( x \) & \( y \) is 7.

(b) Write the negation of the original statement in symbols.

Good: \( \exists x \in \mathbb{R} \) such that, \( \forall \) real numbers \( y \), \( x - y \neq 7 \).

Better: \( \exists x \in \mathbb{R} \mid \forall y \in \mathbb{R}, \ x - y \neq 7 \)

Best: \( \forall y \in \mathbb{R}, \exists x \in \mathbb{R} \mid x - y \neq 7 \).