Summary for Midterm 2

I. First-order systems: General theory.

1. Notion of a solution (curve), equilibrium, phase portrait.
2. Converting 2nd order equations into systems.
3. Vector and direction fields vs. solution curves.
5. Existence and uniqueness of solutions and applications.
6. The harmonic oscillator.

II. Planar linear systems: Methods for solving.

1. Matrix notation. Number of equilibria of a linear system.
2. The linearity principle.
3. Linear independence and linearly independent solutions.
4. Finding the general solution given two independent solutions: \( Y(t) = k_1Y_1(t) + k_2Y_2(t) \).
5. Eigenvalues and eigenvectors of a matrix: \( AV = \lambda V \). The characteristic polynomial \( \text{det}(A - \lambda I) \).
6. Straight-line solutions \( e^{\lambda t}V \).
7. Real distinct eigenvalues: saddles \((\lambda_1 < 0 < \lambda_2)\), sinks \((\lambda_1, \lambda_2 < 0)\), and sources \((\lambda_1, \lambda_2 > 0)\).

III. Changing variables and linearization for 1st order equations. Qualitative analysis.