Solve the given initial value problem:
\[
\frac{dy}{dt} = \frac{t^2}{y + t^3}, \quad y(0) = -2.
\]

**SOLUTION:** Separating the variables and integrating, we obtain
\[
\int y\,dy = \int \frac{t^2}{1 + t^3}\,dt.
\]
The right-hand side can be solved using the substitution \( u = 1 + t^3 \). Then \( du = 3t^2\,dt \), so
\[
\frac{y^2}{2} = \frac{1}{3} \log |1 + t^3| + C.
\]
Therefore,
\[
y(t) = \pm \sqrt{\frac{2}{3} \log |1 + t^3| + K}.
\]
where \( K = 2C \) is a constant. The initial condition \( y(0) = -2 \) gives us
\[
-2 = \pm \sqrt{\frac{2}{3} \log 1 + K} = \pm \sqrt{K}.
\]
Therefore, we must choose the **negative** solution and \( K = 4 \). It follows that the solution to the given initial value problem is
\[
y(t) = -\sqrt{\frac{2}{3} \log(1 + t^3) + 4}.
\]

**Remark.** Observe \( y(t) \) is only defined for \( t > -1 \).