Sec. 2.3, ex 3. Suppose \((x_m)\) has two distinct limits, \(x_0\) and \(y_0\). Choose disjoint neighborhoods \(U\) of \(x_0\) and \(V\) of \(y_0\). For instance, take \(U = B(x_0, \varepsilon)\) and \(V = B(y_0, \varepsilon)\), where \(0 < \varepsilon \leq |x_0 - y_0|/2\). Show that all but finitely many \(x_m\)'s are in both \(U\) and \(V\), which is impossible.

Sec. 2.3, ex. 5 Hint 1: If \(x_0 \in A\), there’s nothing to prove, since \(A \subset \text{cl } A\). So assume \(x_0 \notin A\). Let \(U\) be an arbitrary neighborhood of \(x_0\). Show that \(U\) contains both elements of \(A\) and elements of \(A^c\). This proves that \(x_0 \in \partial A\).

Hint 2: Let \(U\) be an arbitrary neighborhood of \(x_0\). Show that for some \(N\), \(X = \{x_N, x_{N+1}, \ldots\} \subset U\). If \(X\) is finite (for some \(U\)), then the sequence is eventually constant so \(x_0 = x_\ell\) (for some \(\ell\)), hence \(x_0 \in A \subset \text{cl } A\). If \(X\) is infinite for every \(U\), then \(x_0\) is an accumulation point of \(A\).

Sec. 2.4, ex. 4 (a) Assume \(A\) has no isolated points. Then every point of \(A\) is an accumulation point of \(A\), so \(A \subset A'\). Therefore, \(\text{cl } A = A \cup A' = A'\).

(b) Suppose \(A\) is dense in \(B\) and \(B\) is dense in \(C\). Let \(z \in C\) be arbitrary and let \(U\) be an arbitrary neighborhood of \(z\). We need to show that \(U \cap A\) is infinite. Since \(B\) is dense in \(C\), \(U \cap B\) is infinite. Pick \(y \in U \cap B\). Let \(V\) be a neighborhood of \(y\) contained in \(U\). Use the assumption that \(A\) is dense in \(B\) to complete the proof.