San José State University

Math 70, Fall 2009

Sample Final Exam

Name:

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1. Consider the following system of linear equations:

\[
\begin{align*}
x_1 - x_2 &= 4 \\
3x_1 + kx_2 &= 7.
\end{align*}
\]

(a) For which $k$ does the system have a unique solution?

(b) For which $k$ is the system inconsistent?

(c) Are there values of $k$ for which the system has infinitely many solutions?

Solution:
2. Find $a, b, c,$ and $d$ so that

\[
\begin{bmatrix}
1 & -2 \\
2 & -3
\end{bmatrix}
\begin{bmatrix}
a & b \\
c & d
\end{bmatrix} = \begin{bmatrix}
1 & 0 \\
3 & 2
\end{bmatrix}.
\]

Solution:
3. Urn 1 contains 1 white and 4 red balls, and Urn 2 has 3 white and 2 red balls. An urn is chosen at random, with one as likely to be chosen as the other. Then a ball is withdrawn from the chosen urn.

(a) If a red ball is drawn, what is the probability that it came from Urn 2?

(b) If a white ball is drawn, what is the probability that it came from Urn 1?

Solution:
4. In a group of $n$ people ($n \leq 100$), each person is asked to select a number between 1 and 100, write a number on a slip of paper, and place the slip in a hat. What is the probability that at least 2 of the slips in the hat have the same number written on them?

Solution:
5. Solve the given linear programming problem using a corner point table:

Maximize and minimize  \( P = 10x_1 + 30x_2 \)

subject to

\[
\begin{align*}
2x_1 + x_2 & \geq 16 \\
x_1 + x_2 & \geq 12 \\
x_1 + 2x_2 & \geq 14 \\
x_1, x_2 & \geq 0.
\end{align*}
\]

Solution:
6. You are given the following linear programming problem:

Maximize \( P = 15x_1 + 10x_2 \)

subject to

\[
\begin{align*}
2x_1 + x_2 & \leq 10 \\
x_1 + 3x_2 & \leq 10 \\
x_1, x_2 & \geq 0.
\end{align*}
\]

(a) Using slack variables, write the initial system.

(b) Write the initial simplex tableau, circle the first pivot, and identify the entering and exiting variables.