Abstract: In a first course in linear algebra, the basic theorems on bases and dimensions of vector spaces are covered only in the case of finite dimensional vector spaces. We will show how this looks without the assumption of finite dimensionality, and how it depends on the surprisingly simple arithmetic of infinite cardinal numbers. Our proof of the existence of bases uses the axiom of choice, which we will briefly explain.