1. Let $\gamma : I \to \mathbb{R}^3$ (where $I \subset \mathbb{R}$ is an interval) be a parametrized curve and let $v \in \mathbb{R}^3$ be a fixed vector. Assume that $\gamma'(t)$ is orthogonal to $v$ for all $t \in I$ and that $\gamma(0)$ is also orthogonal to $v$. Prove that $\gamma(t)$ is orthogonal to $v$ for all $t \in I$.

2. Let $\gamma : I \to \mathbb{R}^3$ be a parametrized curve with $\gamma'(t) \neq 0$ for all $t \in I$. Show that $\|\gamma(t)\|$ is a nonzero constant if and only if $\gamma(t)$ is orthogonal to $\gamma'(t)$ for all $t \in I$.

3. Let $\gamma(t) = (ae^{bt}\cos t, ae^{bt}\sin t)$ ($t \in \mathbb{R}$) be a parametrized curve, where $a > 0$, $b < 0$ constants. Show that $\gamma'(t) \to (0, 0)$, as $t \to +\infty$ and that

$$\lim_{t \to \infty} \int_{t_0}^{t} \|\gamma'(s)\| \, ds$$

is finite. That is, $\gamma$ has finite arc-length in $[t_0, \infty)$.

4. Exercise 1.2.4 from Pressley.

5. Let $\gamma : (0, \pi) \to \mathbb{R}^2$ be given by

$$\gamma(t) = \left( \sin t, \cos t + \log \tan \frac{t}{2} \right),$$

where $t$ is the angle that the $y$-axis makes with the vector $\gamma'(t)$. The trace of $\gamma$ is called the tractrix. See Fig. 1. Show that:

(a) $\gamma$ is a differentiable curve, regular except at $t = \pi/2$.

(b) The length of the segment of the tangent of the tractrix between the point of tangency and the $y$-axis is constantly equal to 1.

![Figure 1. The tractrix.](image-url)