Exercise 2.2.6: Let $x$ be a fixed point of a map $f$ on the real line such that $|f'(x)| = 1$ and $f''(x) \neq 0$. Show that arbitrarily close to $x$ there is a point $y$ such that the iterates of $y$ do not converge to $x$.

Problem 2.2.13: Suppose that $I$ is a closed bounded interval and $f : I \to I$ is such that $d(f(x), f(y)) < d(x, y)$ for any $x \neq y$ (this is weaker than the assumption of the Contraction Principle). Prove that $f$ has a unique fixed point $x_0 \in I$ and that $\lim_{n \to \infty} f^n(x) = x_0$ for any $x \in I$.

Problem 2.2.14: Show that the assertion of the previous exercise is not valid for $I = \mathbb{R}$ by constructing a map $f : \mathbb{R} \to \mathbb{R}$ such that $d(f(x), f(y)) < d(x, y)$ for $x \neq y$, $f$ has no fixed point, and $d(f^n(x), f^n(y))$ does not converge to zero for some $x, y$. 