Exercise 7.2.5: Suppose $f, g : X \to X$ are continuous maps of a compact space $X$ and $h : X \to X$ is a topological conjugacy between $f$ and $g$ ($hf = gh$). Assume $f$ has sensitive dependence on initial conditions with sensitivity constant $\Delta_f$; let us show that $g$ has it too.

Define
\[ \Delta_g = \min\{d(h(u), h(v)) : d(u, v) \geq \Delta_f\}. \]

Since $X$ is compact and $h$ is continuous, $\Delta_g$ exists and is positive. We claim that $\Delta_g$ is a sensitivity constant for $g$.

Let $y \in X$ and $\epsilon > 0$ be arbitrary. Continuous maps of compact spaces are uniformly continuous, so there exists $\delta > 0$ such that $d(p, x) < \delta$ implies $d(h(p), h(x)) < \epsilon$. Set $x = h^{-1}(y)$. Since $f$ has sensitive dependence, there exists a point $p$ and a natural number $N$ such that $d(p, x) < \delta$ and $d(f^N(x), f^N(p)) \geq \Delta_f$. Let $q = h(p)$; then $d(g, y) < \epsilon$. By definition of $\Delta_g$ and the fact that $hf = gh$, it follows that
\[ d(g^N(y), g^N(q)) = d(hf^N(x), hf^N(p)) \geq \Delta_g. \]

Therefore, $\Delta_g$ is a sensitivity constant for $g$ so $g$ does have sensitive dependence. Thus sensitive dependence is a topological invariant on compact spaces. \qed

Exercise 7.3.1: Let $x \in [0, 1]$ (mod 1). The even $E_2$-iterates of $x$ are in the left half of the unit interval iff all even digits in the binary expansion of $x$ are zero, i.e., if
\[ x = 0.x_10x_30x_5\cdots. \]

The orbit of $x$ is non-periodic iff the sequence $x_1x_3x_5\cdots$ is non-periodic, which is the case, for example, if $0.x_1x_3x_5\cdots$ (base 2) is irrational. So to obtain the desired number $x$, pick an irrational binary number and insert 0's into all the even binary places. \qed

Exercise 7.3.11: Suppose there exists a homeomorphism $h : [0, 1] \to [0, 1]$ such that $hf_4 = f_\lambda h$, for some $0 < \lambda < 4$. The boundary of $[0, 1]$ has to be mapped to itself by $h$, so $h(0), h(1) \in \{0, 1\}$. On the other hand, $h$ maps fixed points of $f_4$ to fixed points of $f_\lambda$, so $h(0) = 0$. It follows that $h(1) = 1$. This yields $1 = h(1) = hf_4(1/2) = f_\lambda h(1/2)$, which is a contradiction, since $f_\lambda(x) < 1$, for all $x$. \qed