**MATH 231A, SPRING 2010**  
**POTENTIAL PROJECT TOPICS**

Here are some potential topics for the Math 231A project.

**L\(^p\) spaces:** For \( p > 0 \) and a measurable set \( E \), \( f \in L^p(E) \) if \( \int_E |f|^p \, dm < \infty \). You would need to discuss the basic properties of \( L^p \) spaces (including the space \( L^\infty \) of essentially bounded functions) and relations between them.

**The Fourier transform:** The Fourier transform is a linear map from \( L^2 \) to itself, which was invented for solving linear PDEs. You would need to talk about the basic properties of the Fourier transform, its inverse and applications to PDEs.

**Hausdorff measure and dimension:** Hausdorff invented a way of measuring the size of sets (so called fractals) whose “dimension” is not an integer. For example, the Cantor set has (Hausdorff) dimension \( \log 2 / \log 3 \). You would need to define the notion of Hausdorff measure and dimension, discuss their basic properties and describe several examples of fractal sets.

**Ergodic theorems:** Ergodic theorems describe the behavior of averages of a function \( f : X \to \mathbb{R} \) along the orbits \( x, T(x), T^2(x), \ldots \) of a transformation \( T : X \to X \), where \( X \) is a measure space. You would need to introduce the set-up (including saying something about abstract measure spaces), state several ergodic theorems (cf., the textbook), discuss their meaning and give several examples.

**Harmonic functions:** A function is called harmonic if its Laplacian equals zero. You would need to discuss the fundamental properties of harmonic functions (esp. the maximum principle), the Dirichlet problem and give some examples.

**History of Lebesgue integration:** Explore the motivation and background for Lebesgue theory, especially the plethora of puzzling examples and open questions in analysis (and applications) in the second half of the nineteenth century.

**Cantor sets:** A Cantor set is a perfect totally disconnected set. Every two Cantor sets are homeomorphic to each other. You would need to discuss this result, talk about where and how Cantor sets appear in mathematics (e.g., dynamical systems), and give some examples.

There are of course many other possibilities.  
I expect you to consult a couple of references. The paper should be at least several pages long and written in \LaTeX.  
Please let me know which topic you want by **March 1**. I would like to receive the first draft of the paper by **May 12**.