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What is...? seminar
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The Burgeoning Calculus of Polyominoes

May 3, 2010, 3 PM
in MH 235

This is particularly true for regions defined on regular lattices such as square, hexagonal or triangular lattices of the plane. On the other hand, many basic parameters associated with closed regions are represented by surface integrals. For instance, the area $A$, center of gravity $\text{CG}$, moment of inertia $I$, of a closed region are defined by the double integrals:

$$A = \iint \frac{1}{m} \, dx \, dy,$$

$$\text{CG} = \left( \bar{x}, \bar{y} \right) = \frac{\int\int x \, dx \, dy}{A}, \frac{\int\int y \, dx \, dy}{A},$$

$$I = \iint (x - \bar{x})^2 + (y - \bar{y})^2 \, dx \, dy - \left( \bar{x}^2 + \bar{y}^2 \right) \frac{A}{m}.$$

In this paper we restrict the study to regions that are commonly used in discrete geometry, namely the polyominos, but one should keep in mind that a more general formulation could be presented. A polyomino is a finite union of closed cells in the unit lattice square (pixels) of the plane whose boundary consists of a simple closed polygonal path (see Fig. 1(a)). In particular, our polyominos are simply connected (contain no holes), and have no multiple points (see Fig. 1(b)). The polygonal path (contour) of a polyomino is encoded by an ordered pair $(s, w)$ where $s$ is a lattice point belonging to and $w$ is a word over the 4-letter alphabet $A = \{r, u, l, d\} = \{r: \rightarrow, u: \uparrow, l: \leftarrow, d: \downarrow\}$ also known as the Freeman chain code, where the letters correspond to the unit translations in the lattice directions: right, up, left and down. The word $w$ represents the perimeter of the polyomino read in a counterclockwise way starting from the point $s$. The use of $s$ may be avoided in the encodings by assuming that $s$ is always the lowest left most point of the polyomino and that $s = (0, 0)$ by using a suitable translation. In this way, the polyomino of Fig. 1(a) is encoded by the single word $w = rrddrurruurdruururullldluulull dddldldd$. 


Fig. 1. (a) A typical polyomino; (b) a closed curve but not a polyomino.