3.1 #6: A forest of order n with c components has $n-c$ edges.

Proof. Add $c-1$ edges to connect among the c components to form a new connected graph G, which is connected without cycle. Hence G is a tree, and so has $n-1$ edges. Therefore the original forest has $n-1-(c-1)=n-c$ edges.

3.1 #14: Let G be a connected graph such that $G-e$ is a tree for some edge $e \in E(G)$. Then G is unicylic.

Proof. Let the order of G be n. Since $G-e$ is a tree, it has $n-1$ edges. Now G must have a cycle, otherwise G is a still a tree on n vertices and n edges, contradiction! Furthermore, G has exactly one cycle, otherwise G has at least two different cycles, both containing the edge $e=uv$. Then there will be two different paths joining u and v (without using the edge uv). It follows that $G-e$ has two different paths joining u and v, contradiction because $G-e$ is a tree.

3.2 #14: If T_1 and T_2 are spanning trees of a connected graph G such that $e_1 \in E(T_1) - E(T_2)$ then there exists $e_2 \in E(T_2)-E(T_1)$ such that $T_2+e_1-e_2$ is also a spanning tree of G.

Proof. By 3.1 #14, T_2+e_1 is unicylic. Let C be the cycle subgraph in T_2+e_1. Then there exists $e_2 \in E(C) \cap E(T_2)$, otherwise C is a subgraph of T_1, contradiction because T_1 is a tree. Now $T_2+e_1-e_2$ is a spanning tree.

3.4 #5: Every even mesh has a perfect matching.

Proof. An even mesh is $P_{k_1} \times \cdots \times P_{k_r}$ where k_i is even for some i. W.L.O.G. we assume k_1 is even. Then P_{k_1} has a perfect matching. The desired result will follow from the Lemma:

Lemma If G has a perfect matching then $G \times H$ also has a perfect matching.

Proof. Let M be a perfect matching of G. Then $M' = \{\langle (g_1,h)(g_2,h)\rangle : g_1g_2 \in M, h \in V(H)\} \subset E(G \times H)$ is a perfect matching for $G \times H$.

3.4 #18: If a tree has a perfect matching, it has only one.

Proof. Suppose that there are trees with at least two distinct perfect matchings. Let T be such tree with the minimum order, and let M_1 and M_2 be two distinct perfect matching on T. Since T is a tree, it has pendent vertex y. Then the pendent edge $xy \in M_1 \cap M_2$. Define $T' = T - \{x,y\}$. Then T' is a forest, and $M_1' = M_1 - \{xy\}$ and $M_2' = M_2 - \{xy\}$ are two distinct perfect matchings for T'. But $|T'| < |T|$ and so T' contains a smaller tree with two distinct perfect matching, hence a contradiction to the minimality of T.