

San Jose State University  
 Department of Mathematics, College of Science  
 Fall 2009  
 MATH 42, Discrete Mathematics, section 1  
 SOLUTION 1

1. (a) False  
 (b) The converse is : if the beach erodes then there is a hurricane.  
 (c)  $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$  is a tautology because the corresponding column in the truth table below has ALL T.

p	q	r	$p \vee q$	$p \rightarrow r$	$q \rightarrow r$	$(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)$	$[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	T	T	T	T	T
T	F	F	T	F	T	F	T
F	T	T	T	T	T	T	T
F	T	F	T	T	F	F	T
F	F	T	F	T	T	F	T
F	F	F	F	T	T	F	T

2. (a) i.  $\forall y L(Jack, y)$   
 ii.  $\forall x \exists y L(x, y)$   
 (b) Take  $x = -100$ . There is NO  $y$  satisfying  $y^2 - (-100) < 100$ .  
 (c) i. False  
 ii. False
3. (a) (1)  $\neg r \rightarrow \neg p$  (premise)  
 (2)  $\neg r$  (premise)  
 (3)  $\neg p$  (modus ponens and (1), (2))  
 (4)  $p \vee q$  (premise)  
 (5)  $q$  (disjunctive syllogism and (3) (4))  
 (b) type of proof: proof by contradiction

Proof: Assume that  $y$  is not odd. Hence  $y$  is even i.e.  $y = 2k$  for some integer  $k$ .  
 Now  $y^3 = (2k)^3 = 8k^3 = 2(4k^3)$ , which is even. It contradicts that  $y^3$  is given as odd.

4. (a) 2  
(b)  $\emptyset, S, \{1\}, \{a\}$   
(c) 27  
(d) 6, 7  
(e) At least 6 and at most 20 people have visited both Russia and China.
5. (a) domain = all positive integers  
range =  $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$   
(b) Yes,  $f$  is onto because given any  $y \in \mathbf{Z}$ , take  $n = 3y \in \mathbf{Z}$  then  $f(n) = f(3y) = \lceil \frac{3y}{3} \rceil = \lceil y \rceil = y$ .  
(c) an example is  $g : \mathbf{N} \rightarrow \mathbf{N}$  defined by  $g(x) = \lfloor \frac{x}{2} \rfloor$
6. (a)  $a_1 = 2, a_2 = 2, a_3 = 3, a_4 = 3, a_5 = 4$   
(b) 14 or  $-14$   
(c)  $a_n = n^3 + 2$