

San Jose State University
Department of Mathematics, College of Science
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MATH 42, Discrete Mathematics, section 2
SOLUTION 1

1. (a) False
 (b) The inverse is : if there is not a hurricane then the beach does not erode.
 (c) $(p \rightarrow r) \wedge (q \rightarrow r)$ and $(p \vee q) \rightarrow r$ are logically equivalent because their corresponding columns in the truth table below are identical.

p	q	r	$p \rightarrow r$	$q \rightarrow r$	$(p \rightarrow r) \wedge (q \rightarrow r)$	$p \vee q$	$(p \vee q) \rightarrow r$
T	T	T	T	T	T	T	T
T	T	F	F	F	F	T	F
T	F	T	T	T	T	T	T
T	F	F	F	T	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	F
F	F	T	T	T	T	F	T
F	F	F	T	T	T	F	T

2. (a) i. $\neg \exists y L(Jill, y)$
 ii. $\forall x \forall y L(x, y)$
 (b) Take $x = 1$ and $y = 0$.
 (c) i. True
 ii. False
3. (a) (1) $\neg r \rightarrow \neg q$ (premise)
 (2) $\neg r$ (premise)
 (3) $\neg q$ (modus ponens and (1), (2))
 (4) $p \vee q$ (premise)
 (5) p (disjunctive syllogism and (3) (4))
 (b) type of proof: proof by contradiction

Proof: Assume that y is not even. Hence y is odd i.e. $y = 2k + 1$ for some integer k . Now $y^3 = (2k + 1)^3 = 8k^3 + 12k^2 + 6k + 1 = 2(4k^3 + 6k^2 + 3k) + 1$, which is odd. It contradicts that y^3 is given as even.

4. (a) 2
(b) $\emptyset, S, \{2\}, \{b\}$
(c) 27
(d) 1, 2, 3, 4, 5, 6, 7, 8, 9
(e) At least 8 and at most 15 people have visited both Russia and China.
5. (a) domain = all positive integers
range = $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
(b) Yes, f is onto because given any $y \in \mathbf{Z}$, take $n = 3y \in \mathbf{Z}$ then $f(n) = f(3y) = \lfloor \frac{3y}{3} \rfloor = \lfloor y \rfloor = y$.
(c) an example is $g : \mathbf{N} \rightarrow \mathbf{N}$ defined by $g(x) = x + 1$
6. (a) $a_1 = 1, a_2 = 2, a_3 = 3, a_4 = 3, a_5 = 4$
(b) 14
(c) $a_n = 3^n + 2$