San Jose State University
Department of Mathematics, College of Science
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MATH 42, Discrete Mathematics, section 2
SOLUTION 2

1. (a) quotient $= -25$, remainder $= 11$
   (b) (proof by contradiction) Suppose that 2?982?4?2242 = (8?8?16)(?4?672) then 2?982?4?2242 $\equiv$ (8?8?16)(?4?672)(mod100), i.e., $42 \equiv 16 \cdot 72 \equiv 52 (\text{mod} 100)$, contradiction.
   (c) 1
   (d) $(1111001101)_2$
   (e) 15 multiplications
2. (a) it has exactly two positive divisors
   (b) 907 (OR 911)
   (c) $3 \cdot 19 \cdot 1627$
   (d) 180
   (e) NO, because 770! has only 7^{127} in its prime factorization
3. (a) $\gcd(321, 2241) = 3$ because

$$\begin{aligned} 2241 &= 321(6) + 315 \\ 321 &= 315(1) + 6 \\ 315 &= 6(52) + 3 \\ 6 &= 3(2) + 0 \end{aligned}$$

   (b) lcm$=2^6 \cdot 3^2 \cdot 5 \cdot 7^3 \cdot 11 \cdot 13^4$
   (c) gcd always divides lcm, but 240 does not divide 2700
4. (a) Let $P(n)$ be the statement that

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

   for $n = 1, 2, 3, \ldots$
   (b) LHS of $P(1) = \frac{1}{1 \cdot 2} = \frac{1}{2}$
   RHS of $P(1) = \frac{1}{1+1} = \frac{1}{2}$
   Hence $P(1)$ is true.

(c) Assume $P(k)$ is true, i.e.,

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{k(k+1)}=\frac{k}{k+1}$$

Consider

$$\begin{aligned}
\text{LHS of } P(k+1) &= \frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{(k+1)(k+1+1)} \\
&= [\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{k(k+1)}]+\frac{1}{(k+1)(k+2)} \\
&= \frac{k}{k+1}+\frac{1}{(k+1)(k+2)} \\
&= \frac{1}{k+1}[k+\frac{1}{k+2}] \\
&= \frac{1}{k+1}\frac{(k+1)^2}{k+2} \\
&= \frac{k+1}{k+2} \\
&= \text{RHS of } P(k+1)
\end{aligned}$$

(d) By the Principle of Mathematical Induction, $P(n)$ is true for all $n = 1, 2, 3, \ldots$.

5. (a) 153
   (b) 48
   (c) TRUE

6. (a) $C(8,3)\ 5^3\ 21^5 = 28588707000$
   (b) $P(8,8)\ P(9,5) = 101606400$
   (c) The 41 picked integers are the pigeons, form 40 pigeonholes by pairing 11 and 90, 12 and 89, etc. Then Pigeonhole Principle guarantees that two of the picked integers will have a sum of 101.