

San Jose State University
Department of Mathematics, College of Science
Fall 2009
MATH 42, Discrete Mathematics
Answers of HW3

Please ask if you do not understand the answers.

Please report if you find any errors, typos.

1.5 # 1

modus ponens,

yes, it is a valid argument

1.5 # 4

a) simplification

b) disjunctive syllogism

c) modus ponens

d) addition

e) hypothetical syllogism

1.5 # 7

modus ponens

1.5 # 14

a) Let $c(x)$ be " x is in this class", $r(x)$ be " x owns a red convertible", and $t(x)$ be " x has gotten a speeding ticket".

Hypotheses: $c(\text{Linda}), r(\text{Linda}), \forall x(r(x) \rightarrow t(x))$

Argument:

Step	Reason
1. $\forall x(r(x) \rightarrow t(x))$	Hypo
2. $r(\text{Linda}) \rightarrow t(\text{Linda})$	universal instantiation using (1)
3. $r(\text{Linda})$	Hypo
4. $t(\text{Linda})$	modus ponens using (2) and (3)
5. $c(\text{Linda})$	Hypo
6. $c(\text{Linda}) \wedge t(\text{Linda})$	conjunction using (4) and (5)
7. $\exists x(c(x) \wedge t(x))$	existential generalization using (6)

Conclusion: $\exists x(c(x) \wedge t(x))$

b) Let $r(x)$ be " x is one of the five roommates listed", $d(x)$ be " x has taken a course in discrete math", and $a(x)$ be " x can take a course in algorithms".

Hypotheses: $\forall x(r(x) \rightarrow d(x)), \forall x(d(x) \rightarrow a(x))$

Argument:

Step	Reason
1. $\forall x(r(x) \rightarrow d(x))$	Hypo
2. $r(y) \rightarrow d(y)$	universal instantiation using (1)
3. $\forall x(d(x) \rightarrow a(x))$	Hypo
4. $d(y) \rightarrow a(y)$	universal instantiation using (3)
5. $r(y) \rightarrow a(y)$	hypothetical syllogism using (2) and (4)
6. $\forall x(r(x) \rightarrow a(x))$	universal generalization using (5)

Conclusion: $\forall x(r(x) \rightarrow a(x))$

- c) Let $s(x)$ be " x is a movie produced by Sales ", $c(x)$ be " x is a movie about coal miners ", and $w(x)$ be " x is a wonderful movie ".

Hypotheses: $\forall x(s(x) \rightarrow w(x))$ and $\exists x(s(x) \wedge c(x))$

Argument:

Step	Reason
1. $\exists x(s(x) \wedge c(x))$	Hypo
2. $s(y) \wedge c(y)$	existential instantiation using (1)
3. $s(y)$	simplification using (2)
4. $\forall x(s(x) \rightarrow w(x))$	Hypo
5. $s(y) \rightarrow w(y)$	universal instantiation using (4)
6. $w(y)$	modus ponens using (3) and (5)
7. $c(y)$	simplification using (2)
8. $w(y) \wedge c(y)$	conjunction using (6) and (7)
9. $\exists x(c(x) \wedge w(x))$	existential generalization using (8)

Conclusion: $\exists x(c(x) \wedge w(x))$

- d) Let $c(x)$ be " x is in this class ", $f(x)$ be " x has been to France ", and $l(x)$ be " x has visited the Louvre ".

Hypotheses: $\exists x(c(x) \wedge f(x))$, $\forall x(f(x) \rightarrow l(x))$

Argument:

Step	Reason
1. $\exists x(c(x) \wedge f(x))$	Hypo
2. $c(y) \wedge f(y)$	existential instantiation using (1)
3. $f(y)$	simplification using (2)
4. $c(y)$	simplification using (2)
5. $\forall x(f(x) \rightarrow l(x))$	Hypo
6. $f(y) \rightarrow l(y)$	universal instantiation using (5)
7. $l(y)$	modus ponens using (3) and (6)
8. $c(y) \wedge l(y)$	conjunction using (4) and (7)
9. $\exists x(c(x) \wedge l(x))$	existential generalization using (8)

Conclusion: $\exists x(c(x) \wedge l(x))$

1.5 # 19

- a) Invalid, fallacy of affirming the conclusion
- b) Valid, modus tollens
- c) Invalid, fallacy of denying the hypothesis

1.6 # 5 direct proof

Since $m + n$ and $n + p$ are even, we have $m + n = 2h$ and $n + p = 2k$ for some integers h and k . Now by adding them together, we have $m + 2n + p = 2h + 2k$, and so $m + p = 2(h + k - n)$. Therefore $m + p$ is even because $h + k - n$ is an integer.

1.6 # 8 proof by contradiction

Assume that $n + 2 = b^2$ is a square. Since $n = a^2$ is a square, we have $2 = (n + 2) - n = b^2 - a^2 = (b - a)(b + a)$. Therefore $b - a = \pm 1$ and $b + a = \pm 2$. It follows that $2b = \pm 3$, contradiction b/c b is an integer.

1.6 # 18

a) by contrapositive

Assume that n is odd. Then $n = 2k + 1$ for some k . Hence $3n + 2 = 3(2k + 1) + 2 = 6k + 5 = 2(3k + 2) + 1$ is odd.

b) by contradiction

Assume the n is odd. Then $n = 2k + 1$ for some k . Hence $3n + 2 = 3(2k + 1) + 2 = 6k + 5 = 2(3k + 2) + 1$ is odd, a contradiction because $3n + 2$ is given as even.

1.6 # 25 by contradiction

Assume that $r = \frac{a}{b}$ is root, where a and b are integers with lowest term. Then $(\frac{a}{b})^3 + \frac{a}{b} + 1 = 0$, and so $a^3 + ab^2 + b^3 = 0$. Now a is even if and only if b is even, but it cannot be the case because $\frac{a}{b}$ is in the lowest term. Therefore both a and b are odd, and so a^3, ab^2, b^3 are all odd. Then $b^3 = -(a^3 + ab^2)$ is even, a contradiction.

1.6 # 38

7 is NOT a sum of squares of three integers.