

San Jose State University
Department of Mathematics, College of Science
Fall 2009
MATH 42, Discrete Mathematics
Answers of HW8

Please ask if you do not understand the answers.

Please report if you find any errors, typos.

5.1 # 3

a) $4^{10} = 1048576$

b) $5^{10} = 9765625$

5.1 # 22

a) 1000

b) 4500

c) 4536

d) 6000

e) 2829

f) 6171

g) 1543

h) 257

5.1 # 30

a) $26^8 = 208827064576$

b) $26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 \cdot 21 \cdot 20 \cdot 19 = 62990928000$

c) $26^7 = 8031810176$

d) $25 \cdot 24 \cdot 23 \cdot 22 \cdot 21 \cdot 20 \cdot 19 = 2422728000$

e) $26^6 = 308915776$

f) $26^6 = 308915776$

g) $26^4 = 456976$

h) $26^6 + 26^6 - 26^4 = 617374576$

5.1 # 35

a) $\begin{cases} 2 & \text{if } n \leq 2 \\ 0 & \text{if } n > 2 \end{cases}$

b) $\begin{cases} 1 & \text{if } n = 1 \\ 2^{n-2} & \text{if } n \geq 2 \end{cases}$

c) $2(n - 1)$

5.1 # 44 Let A be the set of bit strings of length 10 with 5 consecutive 0s, and B be the set of bit strings of length 10 with 5 consecutive 1s.

Then $|A| = |B| = 112$ and $|A \cap B| = 2$

Hence, by inclusion-exclusion principle, $|A \cup B| = |A| + |B| - |A \cap B| = 112 + 112 - 2 = 222$

To see $|A| = 112$, we use the sum rule to divide the set A into six disjoint sets A_5, A_6, A_7, A_8, A_9 and A_{10} , where A_k represents the set of bit strings of length 10 with exactly k consecutive 0s. To count A_5 , we classify them as follows

000001????, 1000001???, ?1000001??, ??1000001?, ???1000001, ????100000

Each ? has two choices, namely 0 or 1. Hence

$$|A_5| = 2^4 + 2^3 + 2^3 + 2^3 + 2^3 + 2^4 = 64.$$

Similarly, we have

$$\begin{aligned} |A_6| &= 2^3 + 2^2 + 2^2 + 2^2 + 2^3 = 28 \\ |A_7| &= 2^2 + 2^1 + 2^1 + 2^2 = 12 \\ |A_8| &= 2^1 + 2^0 + 2^1 = 5 \\ |A_9| &= 2^0 + 2^0 = 2 \\ |A_{10}| &= 1 \end{aligned}$$

Hence $|A| = 1 + 2 + 5 + 12 + 28 + 64 = 112$

To see $|B| = 112$, modify the above argument by interchanging the role of 0 and 1.

5.2 # 4

- a) 5
- b) 13

5.2 # 6 Let us create d pigeonholes called $0, 1, \dots, d-1$. Let the $d+1$ integers be pigeons. When an integer is divided by d , it has a remainder between 0 and $d-1$, and then the integer will go to the pigeonhole according to its remainder. By Pigeonhole Principle, two integers will have same remainder.

5.2 # 9 4951

5.2 # 10 Let us create 4 pigeonholes called EE, EO, OE, OO. Let the 5 ordered pairs of integers be the pigeons. Each pigeon is assigned to a pigeonhole according to the parity of the integers, for example, if x_1 is even and y_1 is odd then (x_1, y_1) is assigned to the pigeonhole EO. By PP ($5 > 4$), there are two ordered pairs of integers (x_i, y_i) and (x_j, y_j) are in the same pigeonhole i.e. x_i and x_j are of same parity, also y_i and y_j are of same parity. Hence both $x_i + x_j$ and $y_i + y_j$ are even and so the mid-point $(\frac{x_i+x_j}{2}, \frac{y_i+y_j}{2})$ is an integer point.

5.2 # 15 4