

San Jose State University
 Department of Mathematics, College of Science
 Fall 2009
 MATH 42, Discrete Mathematics
 Answers of section 4.2

2.4 # 4

- a) 1, -2, 4, -8
- b) 3, 3, 3, 3
- c) 8, 11, 23, 71
- d) 2, 0, 8, 0

2.4 # 5

- a) 2, 5, 8, 11, 14, 17, 20, 23, 26, 29
- b) 1, 1, 1, 2, 2, 2, 3, 3, 3, 4
- c) 1, 1, 3, 3, 5, 5, 7, 7, 9, 9
- d) -1, -2, -2, 8, 88, 656, 4912, 40064, 362368, 3627776
- e) 3, 6, 12, 24, 48, 96, 192, 384, 768, 1536
- f) 1, 1, 2, 3, 5, 8, 13, 21, 34, 55
- g) 1, 2, 2, 3, 3, 3, 3, 4, 4, 4
- h) 3, 3, 5, 4, 4, 3, 5, 5, 4, 3

2.4 # 10 For $n = 1, 2, 3, \dots$

- a) $a_n = n^2 + 2 : \dots, 123, 146, 171$
- b) $a_n = 4n + 3 : \dots, 47, 51, 55$
- c) $a_n =$ binary representation of $n : \dots, 1100, 1101, 1110$
- d) $a_1 = 1, a_2 = 2$ and $a_n = a_{(\lfloor \sqrt{n-1} \rfloor)^2} + a_{(\lfloor \sqrt{n-1} \rfloor - 1)^2} : \dots, 8, 8, 8$
- e) $a_n = 3^{n-1} - 1 : \dots, 59048, 177146, 531440$
- f) $a_n = 1 \cdot 3 \cdots (2n - 3) \cdot (2n - 1) : \dots, 654729075, 13749310575, 316234143225$
- g) $a_n = \frac{1}{2}[1 - (-1)^{\lceil \sqrt{2n+1} \rceil}] : \dots, 0, 0, 0$
- h) $a_n = 2^{2^{n-1}} : \dots, 18446744073709551616, 340282366920938463463374607431768211456, 115792089237316195423570985008687907853269984665640564039457584007913129639936$

2.4 # 19

$$\begin{aligned} \sum_{j=1}^n (a_j - a_{j-1}) &= (a_1 - a_0) + (a_2 - a_1) + (a_3 - a_2) + \cdots + (a_{n-1} - a_{n-2}) + (a_n - a_{n-1}) \\ &= \cancel{a_1} - a_0 + \cancel{a_2} - \cancel{a_1} + \cancel{a_3} - \cancel{a_2} + \cdots + \cancel{a_{n-1}} - \cancel{a_{n-2}} + a_n - \cancel{a_{n-1}} \\ &= a_n - a_0 \end{aligned}$$

2.4 # 20

$$\begin{aligned}\sum_{k=1}^n \frac{1}{k(k+1)} &= \sum_{k=1}^n \left(\frac{1}{k} - \frac{1}{k+1} \right) \\ &= -\sum_{k=1}^n \left(\frac{1}{k+1} - \frac{1}{(k-1)+1} \right) \\ &= -\left(\frac{1}{n+1} - \frac{1}{0+1} \right) \\ &= 1 - \frac{1}{n+1}\end{aligned}$$