Proof: (vacuous proof)

For \( n \in \mathbb{N} \), \( n \geq 1 \), and so \( |n-1| + |n+1| \geq |n+1| \geq 2 \). Hence \( |n-1| + |n+1| \nless 1 \).

Proof: (vacuous proof)

For \( r \in \mathbb{Q}^+ \), \( r^2 - r + 1 = (r - \frac{1}{2})^2 + \frac{3}{4} \geq \frac{3}{4} > 0 \) and so \( r^2 + 1 r \). Hence \( \frac{r^2+1}{r} \nless 1 \).

Case 3: \( n = 0 \). Then \( \frac{3n^2(n^2+1)}{4} = 1 \) is odd. Hence the implication is true.

Case 2: \( n = 1 \). Then \( \frac{3n^2(n^2+1)}{4} = 9 \) is odd. Hence the implication is true.

Case 3: \( n = 2 \). Then \( \frac{3n^2(n^2+1)}{4} = 36 \) is even, and \( \frac{3n^2(n^2+1)}{4} = 100 \) is even. Hence the implication is true.

Assume that \( 3x+1 \) is even. Then \( 3x+1 = 2k \) for some \( k \in \mathbb{Z} \). Hence \( 5x - 2 = 3x + 2x - 4 + 1 = 2(k + x - 2) + 1 \) is odd because \( k + x - 2 \in \mathbb{Z} \).

Assume that \( 5x - 2 \) is odd. Then \( 5x - 2 = 2k + 1 \) for some \( k \in \mathbb{Z} \). Hence \( 3x + 1 = (5x - 2) - 2x + 3 = 2(k - x + 2) \) is even because \( k - x + 2 \in \mathbb{Z} \).

Assume that \( n \notin \mathbb{A} \). Then \( n \in B = \{2, 3, 6, 7\} \). Hence \( \frac{2^2 + 3(2)-4}{2} = 3, \frac{3^2 + 3(3)-4}{2} = 7, \frac{6^2 + 3(6)-4}{2} = 25 \), and \( \frac{7^2 + 3(7)-4}{2} = 33 \) are all odd.

Case 1: \( n \) is even. Then \( n = 2k \) for some \( k \in \mathbb{Z} \). Hence \( n^3 - n = n(n^2 - 1) = 2k(4k^2 - 1) \) is even \( b/c k(4k^2 - 1) \in \mathbb{Z} \).

Case 2: \( n \) is odd. Then \( n = 2k + 1 \) for some \( k \in \mathbb{Z} \). Hence \( n^3 - n = n(n^2 - 1) = n((2k + 1)^2 - 1) = 2n(2k^2 + 2k) \) is even \( b/c n(2k^2 + 2k) \in \mathbb{Z} \).

Since \( 3x + 4y \) and \( 4x + 5y \) are even, we have \( x = (-5)(3x + 4y) + 4(4x + 5y) \) is also even, and \( y = 4(3x + 4y) - 3(4x + 5y) \) is also even.

the result proved is \( \forall x \in \mathbb{Z}, x \) is even \( \iff 3x^2 - 4x - 5 \) is odd.

however, in the proof, there is a mistake of using the term "converse", instead the term "inverse" should be used.

\( \forall x, y \in \mathbb{Z}, x \) is even or \( y \) is even \( \Rightarrow xy^2 \) is even.

the result attempted is \( \forall x, y \in \mathbb{Z}, x \) is even or \( y \) is even \( \Rightarrow xy^2 \) is even.

and the attempted proof is wrong. Since the role of \( x \) and \( y \) are not the same and so cannot be interchanged by simple relabeling, we cannot use assume that \( x \) is even using WLOG!!!