

San Jose State University
Department of Mathematics, College of Science
Fall 2009
MATH 42, Discrete Mathematics
OLD FINAL

1. (10 pts)

(a) Let $P(x, y)$ mean " $y - x = 4xy$ " where the domain consists of all real numbers.

TRUE or FALSE: $\forall x \exists y P(x, y)$

(b) State the **contrapositive** of the implication: "If two students study together then their GPAs are not equal."

(c) Construct the **truth table** for the proposition $[(p \oplus q) \wedge (q \oplus r)] \rightarrow (p \oplus r)$.

2. (10 pts)

(a) Let the universal set be $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

i. Let $X = \{1, 3, 5, 7, 9\}$, $Y = \{3, 6, 9\}$, and $Z = \{2, 3, 5, 7\}$. List the elements in the set $(Y - X) \cup Z$.

ii. Find **nonempty** subsets H, K and G such that $|H \cup K| = |H \cup G|$ and $|H \cap K| = |H \cap G|$ but $K \neq G$.

(b) Let A and B be two finite sets of 3 elements. How many elements are there in the power set of their Cartesian product, $\mathcal{P}(A) \times \mathcal{P}(B)$?

3. (10 pts)

(a) Let $f : \mathbf{Z} \rightarrow \mathbf{Z}$ defined by $f(x) = \lfloor \frac{x^2-1}{3} \rfloor$. Find the numerical value of $f(4)$.

(b) Is the function $g : \mathbf{N} \times \mathbf{N} \rightarrow \mathbf{Z}$ defined by $g(m, n) = 2m - n$ **onto**? Why?

(c) The first two terms of an **arithmetic progression** are $a_1 = -3$ and $a_2 = 6$. Find the numerical values of the common difference and a_{11} .

4. (10 pts)

(a) Use **Pigeonhole principle** to explain why it is true that if eleven distinct integers from 1 to 20 are chosen then two of them will add up to 21.

(b) Follow the proof format of **Mathematical Induction** to show that

$$2^n + 5^n \leq 7^n$$

for $n = 1, 2, 3, \dots$

Statement: *Let $P(n)$ be ...*

Basic Step: *For $n = 1, \dots$*

Induction Step: *Assume $P(k)$ is ...*

Conclusion: *By the Principle of ...*

5. (10 pts)

- (a) Is the integer $109!$ **divisible** by 10^{26} ? Why?
- (b) Find the **remainder** when the integer $98765432199^3 + 6667778882^5$ is divided by 10.
- (c) Use **Euclidean Algorithm** to compute $\gcd(537, 735)$. Show all your steps.

6. (10 pts)

- (a) Consider the experiment of rolling a die and flipping a coin together. Find the **cardinality** of the sample space.
- (b) For a loaded die, an odd number is 3 times as likely to appear as an even number. What is the probability of having a "5" when the loaded die is rolled?
- (c) What is the probability that a five-card poker hand contains four cards of the same kind?

7. (10 pts)

- (a) List all the 2-**combinations** of the set $\{a, b, c, d\}$.
- (b) Four couples sit down in a row for a picture. How many ways are there that corresponding husbands and wives are sitting together?
- (c) How many bit strings of length 7 are there that have **at least two** 1's?

8. (10 pts)

- (a) Consider the experiment of rolling a fair die two times. Let X be the **minimum** of the two numbers appeared. Find the expected value of the random variable X .
- (b) Suppose that Frida selects a ball by first picking one of two boxes at random and then selecting a ball from this box at random. The first box contains four white balls and three blue balls, and the second box contains two white ball and four blue balls. What is the probability that Frida picked a ball from the second box if she has selected a white ball?

9. (10 pts)

- (a) Give the definition of an **anti-symmetric** relation on a set.
- (b) Define a relation R on the set $\{1, 2, 3, 4, 5\}$ as follows:

$$R = \{(1, 3), (2, 3), (3, 3), (4, 3), (5, 3)\}.$$

Is R anti-symmetric? Explain your answer for full credit.

- (c) Let S be an **equivalence relation** on a set. Suppose that we know (d, b) is a member of S . What are other members of S are known for sure? Why?

10. (10 pts)

- (a) True or False: $\bar{x}(y + z)$ is the **dual** expression of $yz + \bar{x}$.
- (b) Find all Boolean triples (x, y, z) such that $\bar{x}(y + z) = yz + \bar{x}$.
- (c) Find the **sum-of-products** expansion of the Boolean function $F(x, y, z)$ that equals 1 if and only if $\bar{x}(y + z) = yz + \bar{x}$.